Quantization of binding energy of structural solitons and lasing regimes of passive mode-locked fiber lasers

Andrey Komarov*, Konstantin Komarov*, François Sanchez**

* Institute of Automation and Electrometry, Siberian Branch of Russian Academy of Sciences, Acad. Koptyug Pr. 1, 630090 Novosibirsk, Russia
** Laboratoire POMA, Université d’Angers, 2 Bd Lavoisier, 49000 Angers, France
Our congratulations to professor V.E. Zakharov with 70-years anniversary!
Outlines

I. The complex Ginzburg-Landau equation with saturable gain

✓ Complex cubic Ginzburg-Landau equation
✓ Nonlinearities and dispersions of high orders
✓ Competition of equilibrium pulses
✓ Multiple pulse operation, multistability, multihysteresis
✓ Threshold dependence of self-start PML

II. Quantization of binding energy of a soliton pair

✓ Bound structural solitons
✓ Types of binding with 0, \( \pi \), and \( \pi/2 \) – phase differences between pulse peak amplitudes
✓ High-stable noise-proof soliton sequences
✓ Cording of the information by such sequences
✓ Harmonic passive mode-locking through bound solitons
I. The complex cubic Ginzburg-Landau equation

\[ \frac{\partial E}{\partial \zeta} = (D_r + iD_i) \frac{\partial^2 E}{\partial \tau^2} + \left[ \frac{a}{1 + b \int I_d \tau} - \sigma + \left( p + i q \right) |E|^2 \right] \]

Saturated gain

\( E, \zeta, \tau \) are dimensionless field amplitude, coordinate, and time, respectively, 
\( D_r \) is gain-loss dispersion, 
\( D_i \) is group velocity dispersion, 
\( a \) is pump power, \( b \) is saturation parameter, \( I = |E|^2 \), 
\( \sigma \) is linear losses, \( p \) is nonlinearity of losses, \( q \) is Kerr nonlinearity

Steady-state pulse for the Eq. (1) is 
\[ E = E_0 \frac{e^{i \delta \omega \zeta}}{ch^{1+i\alpha} (\beta \tau)} \]

Spectral profile of the steady-state pulse

\[ I_\omega = \frac{\pi^2 I_0}{\alpha \beta^2} \frac{sh \pi \alpha}{ch \pi \alpha + ch \frac{\pi \omega}{\beta v_{ge}}} \]

\( \alpha \) – frequency chirp, 
\( \beta \) – inverse length of a pulse, 
\( \omega \) – detuning from the central carrying frequency

K.P. Komarov, Optics and spectroscopy, 60, pp. 231-234, 1986
The complex cubic Ginzburg-Landau equation

Nonlinear-dispersion parameters $\xi$, $\theta$

$\xi = q/p$

$\theta = D_i/D_r$

$q$ is Kerr nonlinearity/
$p$ is nonlinearity of losses

$D_i$ is group velocity dispersion/
$D_r$ is gain-loss dispersion

Fig. 1. With any initial conditions, for $\xi$, $\theta$ from area 1 the PML (single stationary pulse) is realized. For $\xi$, $\theta$ from area 2 the CW (filling all laser resonator by radiation) is established.

Competition of equilibrium pulses

Three steps of transient process

1. Establishment of equilibrium between pumping and depletion of population inversion
2. Establishment of equilibrium of pulse length $\beta^{-1}$ and frequency chirp $\alpha$
3. Competition of equilibrium pulses with different $I_{0k}$

$$E = \sum_k E_{0k} e^{i(\Lambda_k + i\omega_k)\zeta} c h^{1+i\alpha_k} \left[ \beta_k (\tau - \tau_k) \right]$$

$$\delta\Lambda = \left( 1 - \alpha^2 - 2\theta \alpha \right) \beta^2, \quad \delta\Lambda = \Lambda - G,$$

$$\beta^2 = \left( \frac{p}{1 + \theta^2} \frac{1 + \xi \theta}{2 - \alpha^2} \right) I_0$$

$$\alpha^2 + 3 \frac{1 + \xi \theta}{\xi - \theta} \alpha - 2 = 0$$

Fig. 2. Soliton amplification $\delta\Lambda = \delta\Lambda(I_{0k})$.

1) For $\xi, \theta$ from area 1, $\delta\Lambda$ monotonously increases.
2) For $\xi, \theta$ from area 2, $\delta\Lambda$ monotonously decreases.
The complex Ginzburg-Landau equation

Nonlinear-dispersion parameters $\xi, \theta$ of high orders

Fig. 3. Soliton amplification $\delta A = \delta A(I_{0k})$.
1) For $\xi, \theta$ from area 1 the quantization of intracavity radiation is realized.
   Multiple pulse operation, multistability, multihysteresis.
2) For $\xi, \theta$ from area 2 the threshold self-start of PML is established.
The complex Ginzburg-Landau equation

Saturating nonlinearity of losses $p$

Fig. 4. (a) Dependence of soliton amplification on peak pulse intensity $\delta\Lambda(I_{0k})$.
(b) Transient evolution of multiple pulse operation.
(c) Multistability and hysteresis dependence of number of pulses $N$ on pump power $a$.

The complex Ginzburg-Landau equation

Parasitic frequency-dependent losses

\[
\frac{\partial}{\partial t} E(k,t) = \left[ -k^2 (1 + i\theta) + \frac{1}{1 + (\Gamma k)^2} \right] E(k,t)
\]

Fig. 5. Threshold self-start of PML on intensity of initial radiation.

II. The analysed laser setup

Fig. 6. Schematic representation of the investigated fiber ring laser passively mode locked through nonlinear polarization rotation.
The basic equations

\[ \frac{\partial E}{\partial \zeta} = (D_r + iD_i) \left( \frac{\partial^2 E}{\partial \tau^2} + \frac{a}{1 + b \int I d\tau} + iqI \right) E \]  
(Saturated gain)

\[ E_{n+1} = -\eta \left[ \cos(pI_n + \alpha_0) \cos(\alpha_1 - \alpha_3) + i \sin(pI_n + \alpha_0) \sin(\alpha_1 + \alpha_3) \right] E_n \]

\( E, \zeta, \tau \) are dimensionless field amplitude, coordinate, and time, respectively,
\( D_r \) is the frequency dispersion for the gain-loss,
\( D_i \) is the group velocity dispersion,
\( a \) is the pumping, \( b \) is the saturation parameter
\( q \) is Kerr nonlinearity.
\( \eta \) is the transmission coefficient of the polarizer, \( I = |E|^2 \),
\( \alpha_i \) are orientation angles of phase plates, \( \alpha_0 = 2\alpha_2 - \alpha_1 - \alpha_3, p = \sin(2\alpha_3)/3. \)

Komarrov A., Leblond H., Sanchez F. Phys. Rev. E, 72, pp. 025604(R), 2005

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Fig. 7. (a) CW operation. Threshold self-start of PML: multiple pulse operation. Multistability and multihysteresis. (b) Soliton amplification $\delta \Lambda(I_{0k})$. 

Fig. 8. The single soliton passive mode-locking. The upper right inset in Fig. (a) shows the multiplied soliton pedestal. $a = 1.1$, $q = 2$, $D_i = 0.13$ (anomalous dispersion), $\alpha_0 = 0.2$, $\alpha_1 = -1.64$, $\alpha_3 = 0.2$, $D_r$ is determined by the amplification medium: $D_r = D_{r0} G$, $D_{r0} = 0.085$. 

Temporally (a) and spectrally (b) distributions
Fig. 9. Binding energies $\delta J_k$ of pair bound solitons in steady-states are expressed in relative units (the binding energy of the two solitons ($J_k - J_\infty$) divided by the energy $J_\infty$ of the two solitons removed far away from each other). The laser parameters are the same as in the case of Fig. 8.

Fig. 10. Temporal distributions of intensity $I_k$. (a) First energy level (maximum binding energy $J_1$, minimal distance $d_1$ between solitons, $\delta\phi = \pi$), (b) Second level ($J_2, 2d_1, \delta\phi = 0$), (c) Third level ($J_3, 3d_1, \delta\phi = \pi$).
The minimal losses are realised under antiphase interference between maximum of one soliton and wing of another. As a result, the total phase change of a field on the closed trajectory is \( \delta \phi_1 + \delta \phi_2 = 2\pi k \), were \( k \) is an integer.

The analogy with the condition of an energy quantization for a particle in potential well. The formulation of Bohr’s rule: on the closed trajectory of a movement of a particle, there should be an integer number of de Broglie’s wavelengths.
The antiphase and inphase boundary conditions

Movement of $\pi/2$-phase bound solitons

\[ \begin{align*}
\varphi_1(\tau_{2\text{max}}) &= \varphi_2(\tau_{2\text{max}}) - \pi + 2\pi k_1, \\
\varphi_2(\tau_{1\text{max}}) &= \varphi_1(\tau_{1\text{max}}) + 2\pi k_2 \\
\delta\varphi_1 + \delta\varphi_2 &= 2\pi(k - 1) + \pi \\
\delta\varphi &= \pi k - \pi/2, \quad k = 1, 2, \ldots
\end{align*} \]

Fig. 12. Elastic collision between soliton pair with phase difference equal $\delta\varphi = \pi/2$ and single pulse.
Coding of the information

“Molecular chain” with two types of interval between solitons

Fig. 13. Stable sequence of bound solitons with the first energy level (0) and the second level (1), in which the number 2708 is coded in binary system 101010010100. 

\[ 2708 = 1 \cdot 2^{11} + 0 \cdot 2^{10} + 1 \cdot 2^9 + 0 \cdot 2^8 + 1 \cdot 2^7 + 0 \cdot 2^6 + 0 \cdot 2^5 + 1 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 0 \cdot 2^0. \]

The laser parameters are the same as in the case of Fig. 8.
Coding of the information

1 August 1939

Fig. 14. Stable sequence of bound solitons in which the number 01081939 is coded in binary system as 01000100001001010011.

The laser parameters are the same as in the case of Fig. 8.
Long train of bound solitons

One type of interval between solitons along the train

Fig. 15. Stable sequence of 77 ultrashort pulses.

\[ I \]

\[ \tau \]

\[ a = 2.5 \]
\[ D_i = 0.13 \]
Harmonic passive mode-locking through bound solitons

Fig. 16. Under increasing pump power the train of bound solitons with periodic structure of an ideal crystal fills in all resonator. As a result, harmonic PML with 314 pulses is realized.

By numerical simulation we have found:

- Quantization of binding energy of structural solitons
- Types of binding with 0, $\pi$, and $\pi/2$ – phase differences between pulse peak amplitudes
- High-stable noise-proof soliton sequences
- Cording of the information by such sequences
- Harmonic passive mode-locking through bound solitons
Novosibirsk Akademgorodok