Dynamical Toroidal Solitons with Nonzero Hopf Invariant in a Uniaxial Ferromagnet

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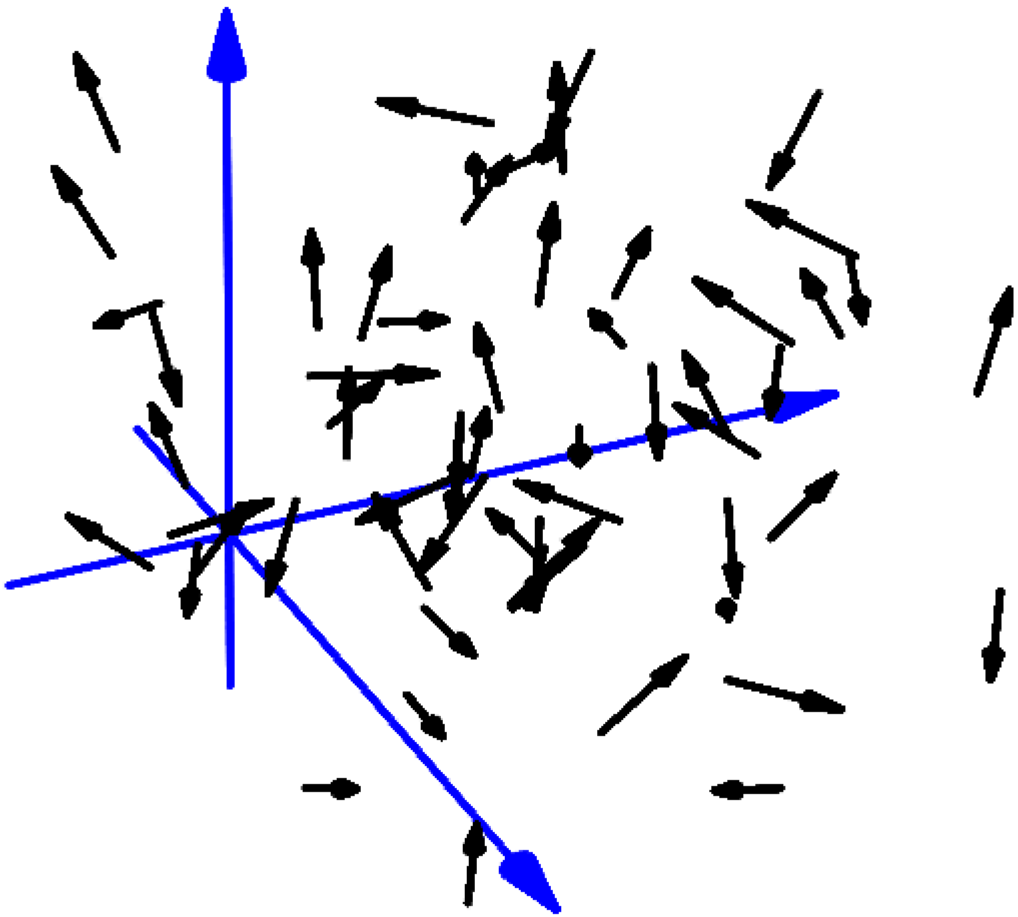
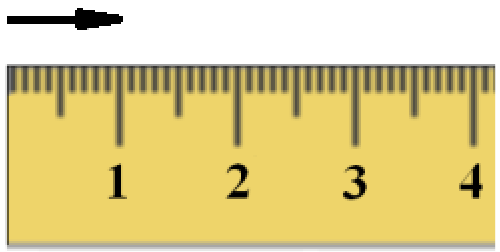
3D unit vector field $\vec{n} = \mathbf{n}(x, y, z), \quad |\mathbf{n}| = 1$

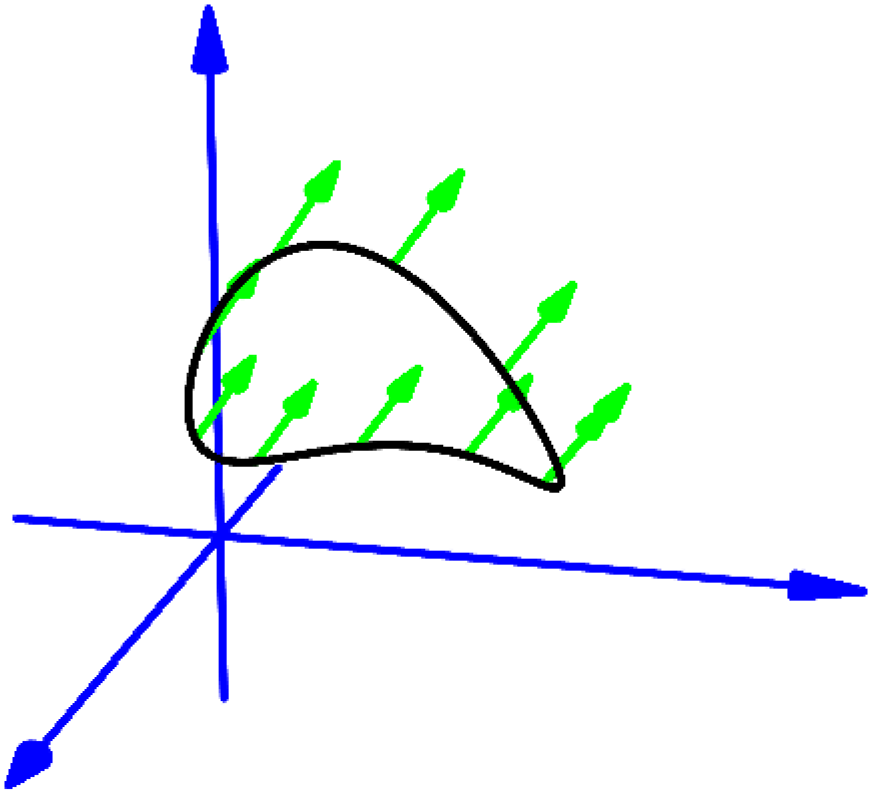


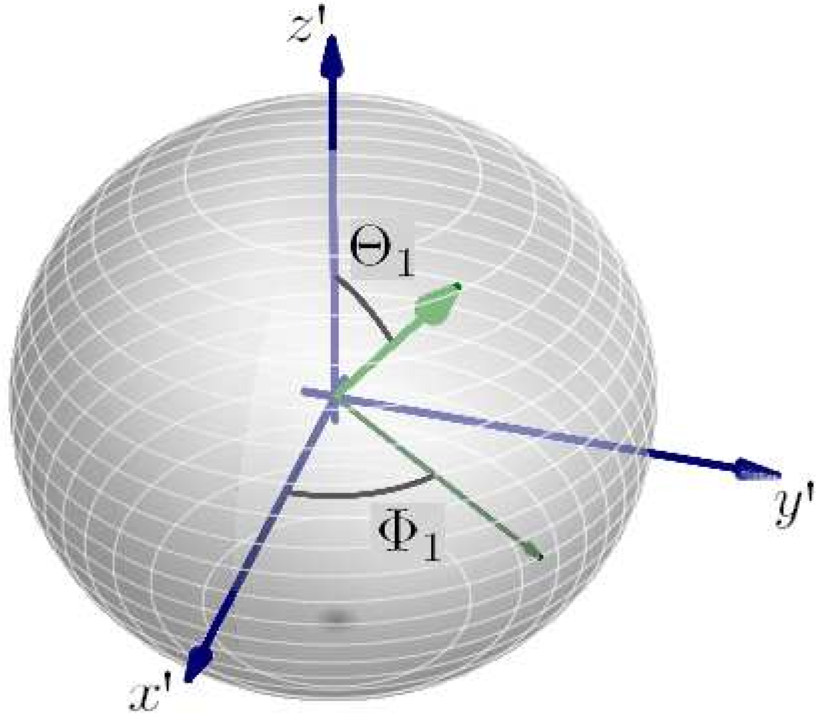


We can choose any vector value:

$$\vec{n}_1 = (sin\Theta_1 cos\Phi_1, sin\Theta_1 sin\Phi_1, cos\Theta_1)$$

curve in **R**^{3} equal to one point on sphere **S**^{2}



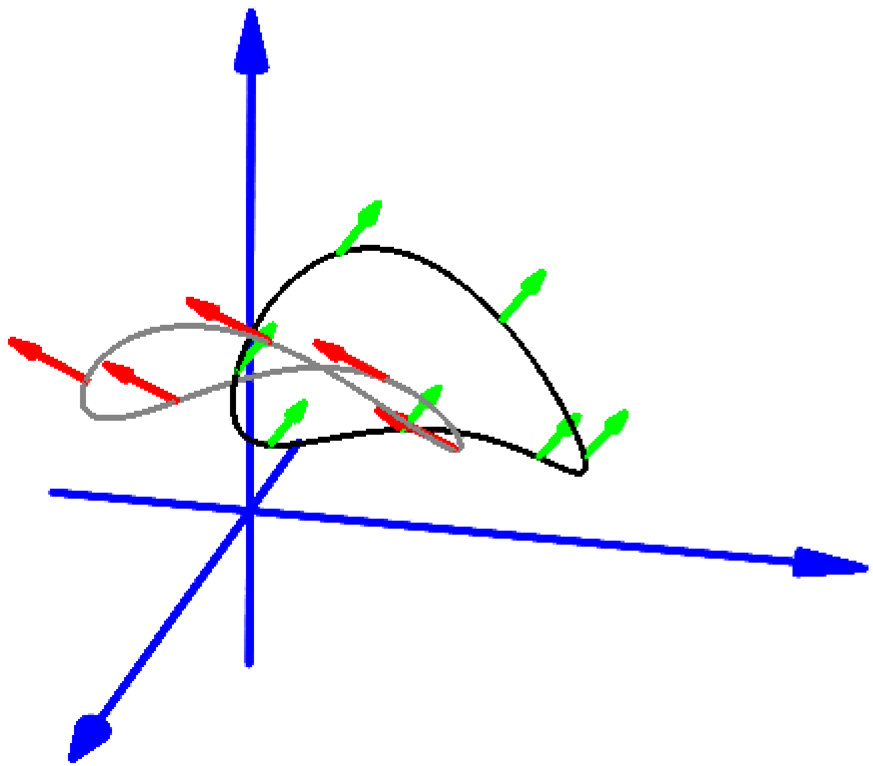


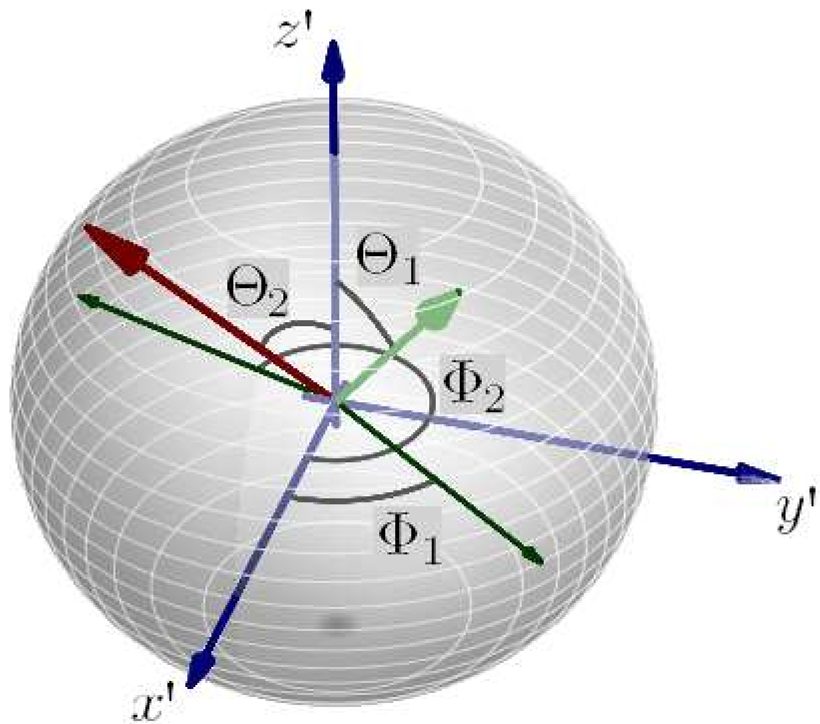
We can choose any 2 vectors:

$$\vec{n}_1 = (sin\Theta_1 cos\Phi_1, sin\Theta_1 sin\Phi_1, cos\Theta_1)$$

$$\vec{n}_2 = (sin\Theta_2 cos\Phi_2, sin\Theta_2 sin\Phi_2, cos\Theta_2)$$

2 curves in $\mathbf{R}^3$ equal to 2 points on sphere $\mathbf{S}^2$





Let $\vec{n} \to (0,0,1)$ (to ground state) as $|\vec{r}| \to \infty$

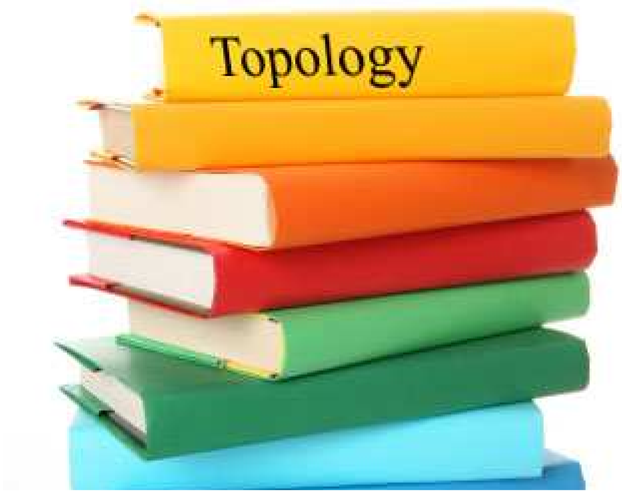
Maps: $\mathbb{R}^3 \cup \{\infty\} \to \mathbb{S}^2$ are classified by a homotopy invariant:

Hopf number $$H = -\frac{1}{(8\pi)^2} \int \mathbf{F} \cdot \mathbf{A} d\mathbf{r}$$

where

$$F_i = \epsilon_{ijk} \mathbf{n} \cdot (\nabla_j \mathbf{n} \times \nabla_k \mathbf{n}), \quad curl\, \mathbf{A} = 2\mathbf{F}$$

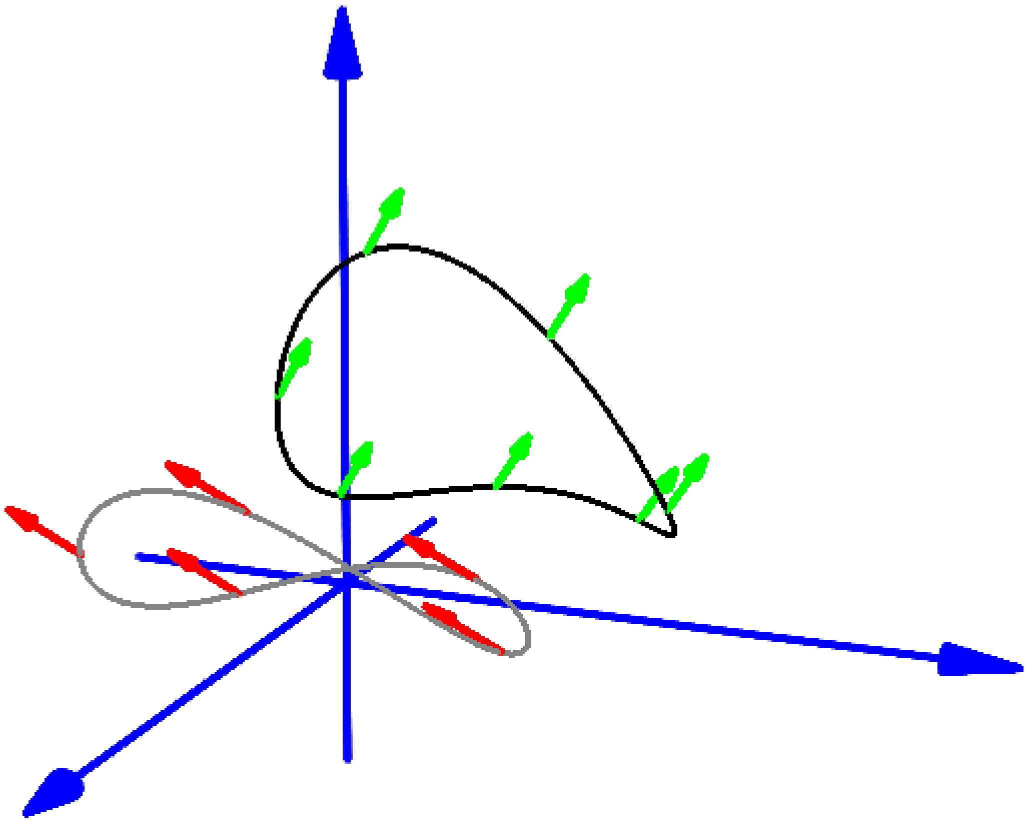
Hopf number takes only integer values: $H \in \mathbb{Z}$



Hopf number = linking number of 2 preimage (in $\mathbf{R}^3$) of 2 points on $\mathbf{S}^2$

Example №1

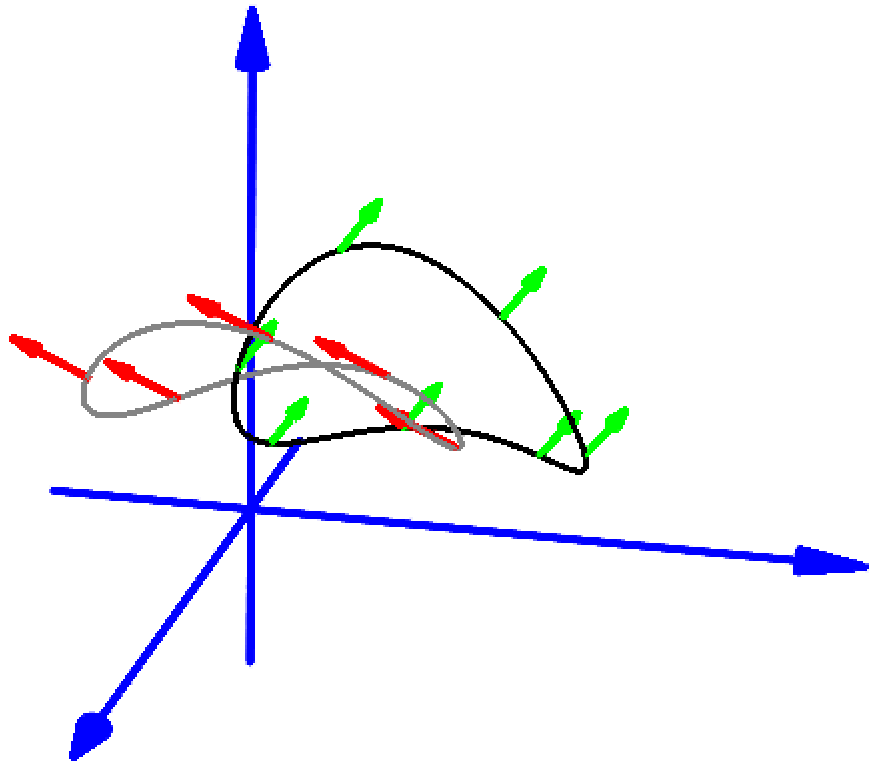
H=0



Hopf number = linking number of 2 preimage (in **R**^{3}) of 2 points on **S**^{2}

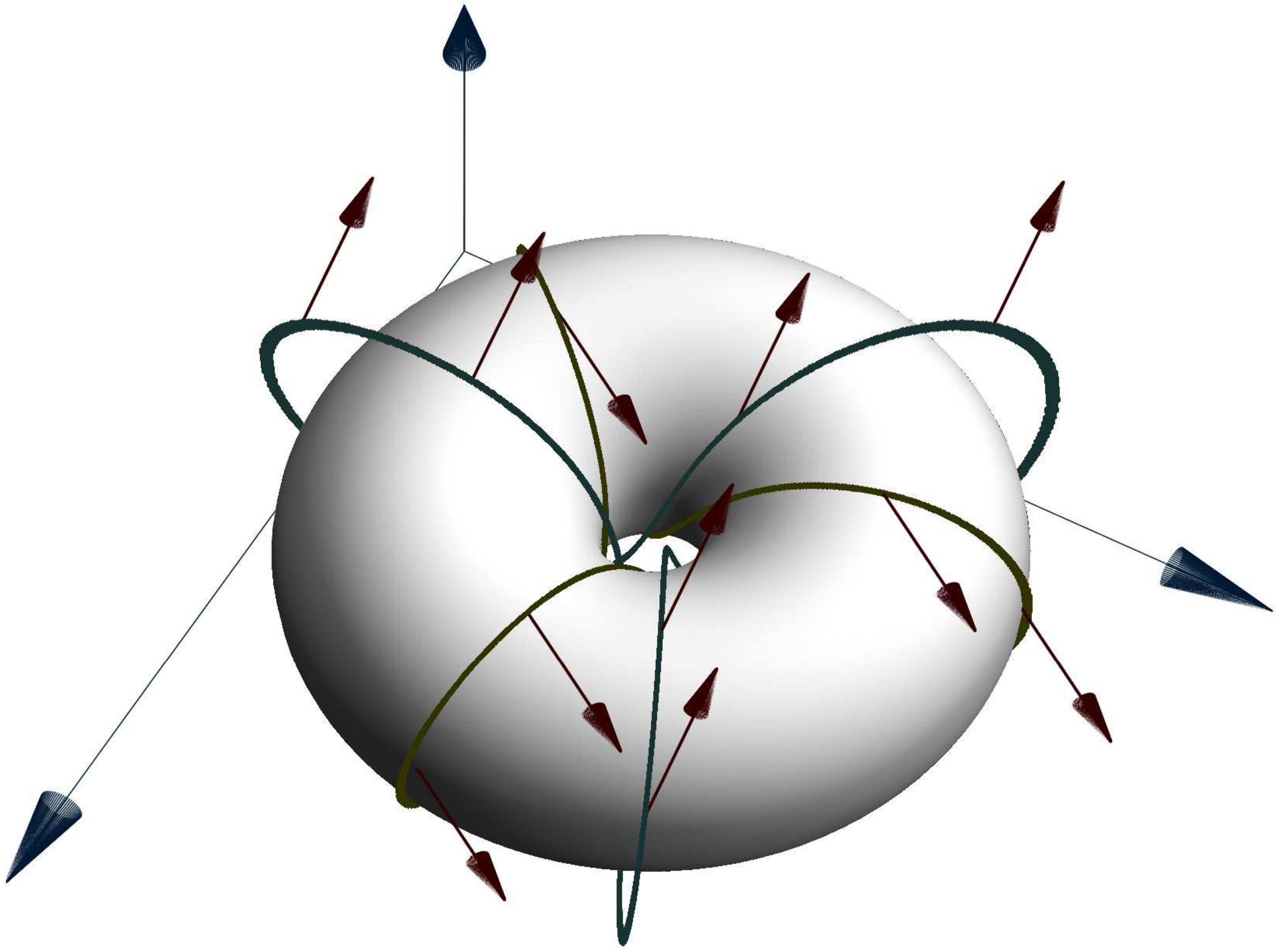
Example №2

H=1

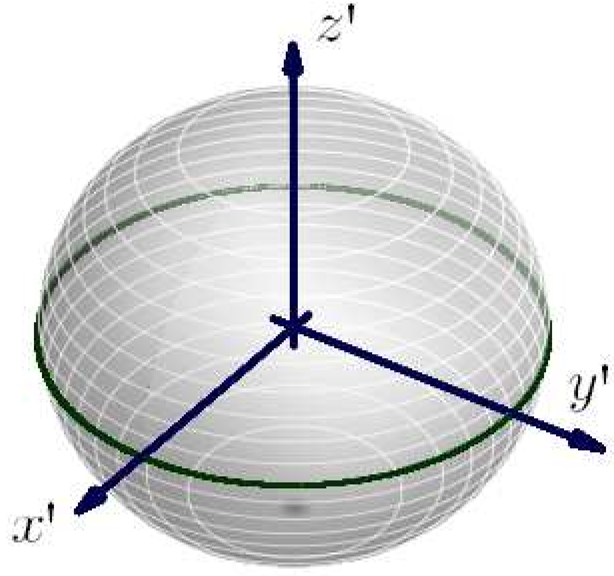


Example №3: field structure of toroidal Hopfion

***H*=3**



Torus in **R³** is the preimage of equator line on **S²**:



FERROMAGNETIC

Landay-Lifshitz equation

$$\frac{\partial \mathbf{M}}{\partial t} = -\gamma \left[\mathbf{M} \times \mathbf{H}_{\mathbf{eff}} \right], \quad \mathbf{H}_{\mathbf{eff}} = -\frac{\delta E}{\delta \mathbf{M}}$$

where

Magnetization vector

$$\mathbf{M} = M_0 \mathbf{n} = M_0 (sin\Theta cos\Phi, sin\Theta sin\Phi, cos\Theta)$$

Energy

$$E = \frac{\alpha}{2} \int (\partial_i \mathbf{M})^2 \, d\mathbf{r} + \frac{\beta}{2} \int ({M_x}^2 + {M_y}^2) \, d\mathbf{r}$$

What do we know about hopfions in ferromagnets ?

Stationary precession toroidal hopfions

- [I.E. Dzyloshinskii, B.A. Ivanov, JETP Lett. 29, 540 (1979)] – first announcement.

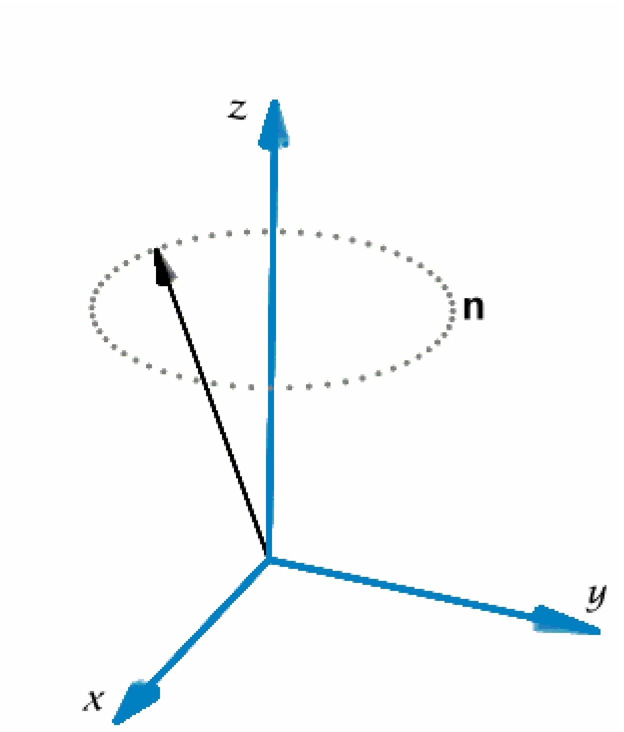
- [A.M. Kosevich, B.A. Ivanov, A.S. Kovalev, Nonlinear waves of magnetisation. Dynamical and topological solitons (book in Russian, 1983) and Phys. Rep. 194, 117 (1990)] – simple analysis and conclusion: only one way to get solution - to use numerical methods.

- [A.B. Borisov, F.N. Rybakov, JETP Lett. 88, 264 (2008)] – hopfions are found numerically. Fine structure and main features are studied.

Why "precession" ?

Hobart-Derrick theorem forbids the existence stationary 3D solitons with energy

$$E = \frac{\alpha}{2} \int (\partial_i \mathbf{M})^2 \, d\mathbf{r} + \frac{\beta}{2} \int \left({M_x}^2 + {M_y}^2 \right) d\mathbf{r}$$



Moving precession toroidal hopfions

- [N.R. Cooper, Phys. Rev. Lett. 82, 1554 (1999)] – announcement.
- [P. Sutcliffe, Phys. Rev. B. 76, 184439 (2007)] – attempt to solve problem by numerical methods, unsuccessful.
- [A.B. Borisov, F.N. Rybakov, work in preparation] – our actual results.

We seek solution, describing precession uniformly moving along anisotropy axis toroidal hopfions of the form:

$$\Phi = \omega t + Q\varphi + \phi(r, z - Vt), \quad \Theta = \theta(r, z - Vt)$$

in cylindrical coordinate system (r, φ, z)

$$Q \in \mathbb{Z} \setminus \{0\}, \; T = \frac{1}{4\pi} \int_{-\infty}^{\infty} \int_{0}^{\infty} sin\theta \left(\partial_r\theta\partial_z\phi - \partial_z\theta\partial_r\phi\right) drdz, \; T = 1$$

For such solitons Hopf index $H = QT = Q$

Analytical approach. Two dimensional elliptic problem:

$$\frac{1}{\alpha\gamma M_0}(\omega - V\partial_z\phi) - sin\theta cos\theta\left(\frac{Q^2}{r^2} + \frac{\beta}{\alpha} + (\partial_r\phi)^2 + (\partial_z\phi)^2\right) + \frac{1}{r}\partial_r\theta + \frac{\partial^2\theta}{\partial r^2} + \frac{\partial^2\theta}{\partial z^2} = 0$$

$$\frac{1}{\alpha\gamma M_0}V\partial_z\theta + 2cos\theta\left(\partial_r\theta\partial_r\phi + \partial_z\theta\partial_z\phi\right) + sin\theta\left(\frac{1}{r}\partial_r\phi + \frac{\partial^2\phi}{\partial r^2} + \frac{\partial^2\phi}{\partial z^2}\right) = 0$$

Numerical approach. Energy minimization:

$$E = \alpha {M_0}^2 \int_{-\infty}^{\infty}\int_0^{\infty}\left[(\partial_r\mathbf{n})^2 + (\partial_z\mathbf{n})^2 + \left(\frac{Q^2}{r^2} + \frac{\beta}{\alpha}\right)({n_x}^2 + {n_y}^2)\right]\pi r dr dz \rightarrow min$$

with constraints:

$${n_x}^2 + {n_y}^2 + {n_z}^2 = 1$$

$$N = \frac{M_0}{\gamma\hbar}\int_{-\infty}^{\infty}\int_0^{\infty}(1 - n_z)2\pi r dr dz = const_1$$

$$P = -\frac{M_0}{\gamma}\int_{-\infty}^{\infty}\int_0^{\infty}\mathbf{n}\cdot\left[\partial_r\mathbf{n}\times\partial_z\mathbf{n}\right]\pi r^2 dr dz = const_2$$

We used specially made algorithm based on conjugate gradient minimization method.

Each of the calculation process takes about 4 hours of CPUs time on a standard server computer equipped with 2 Quad-core Intel Xeon/2.33GHz processors; parallel algorithm with 8 threads, 600x400 grid.

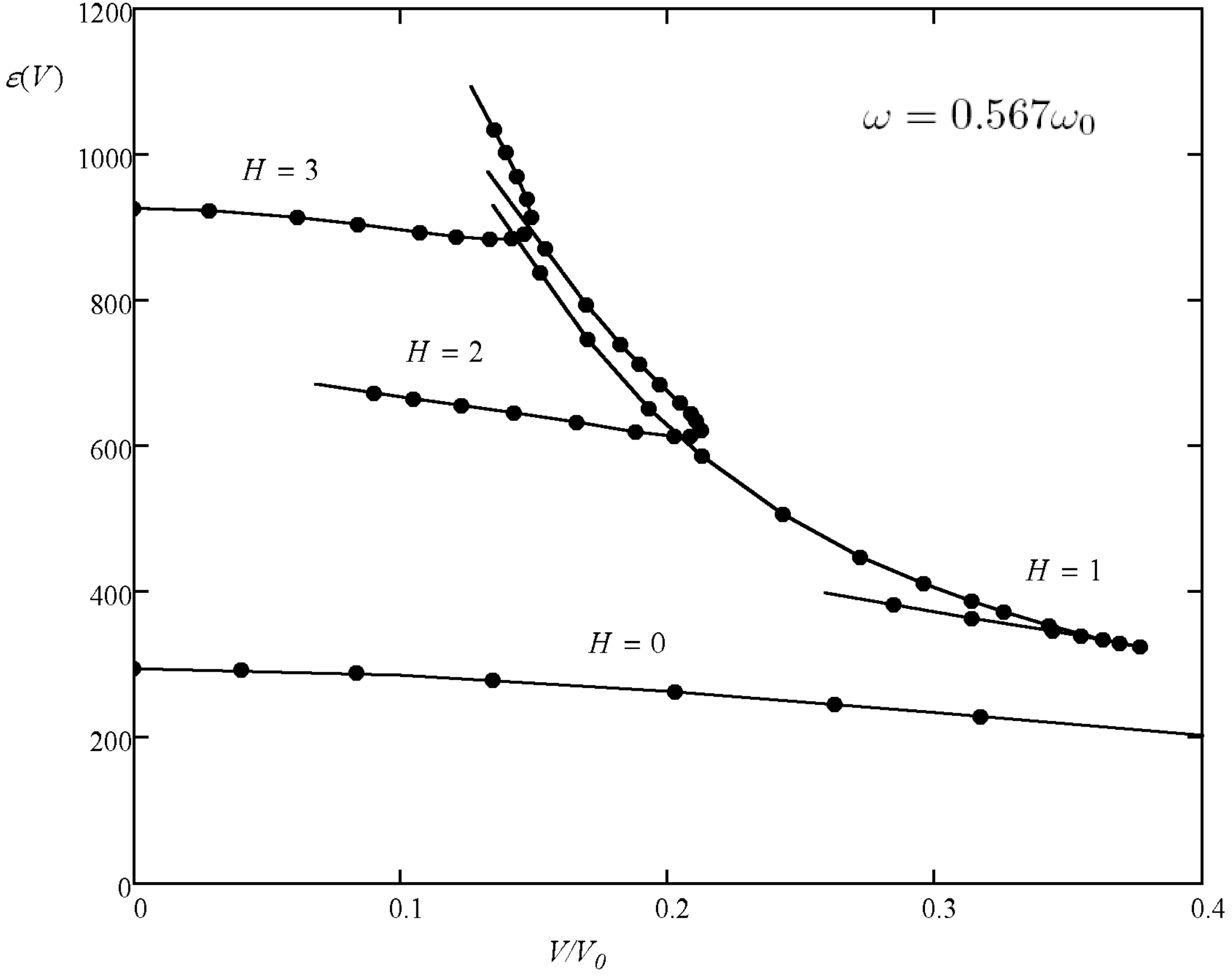
Energy versus the speed for solitons with same precession frequency.

$$\omega_0 = \gamma M_0 \beta$$

$$\varepsilon = E/(\alpha {M_0}^2 l_0)$$

$$V_0 = \gamma M_0 \sqrt{\alpha\beta}$$

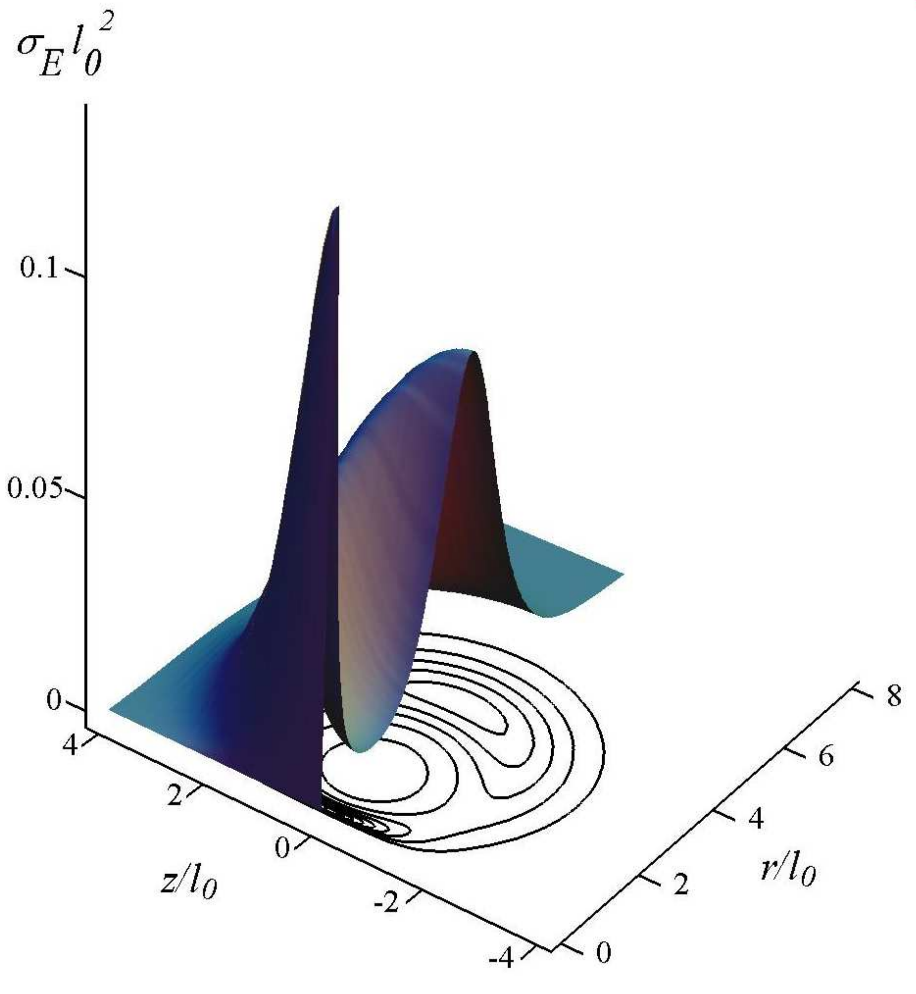
$$l_0 = \sqrt{\alpha/\beta}$$



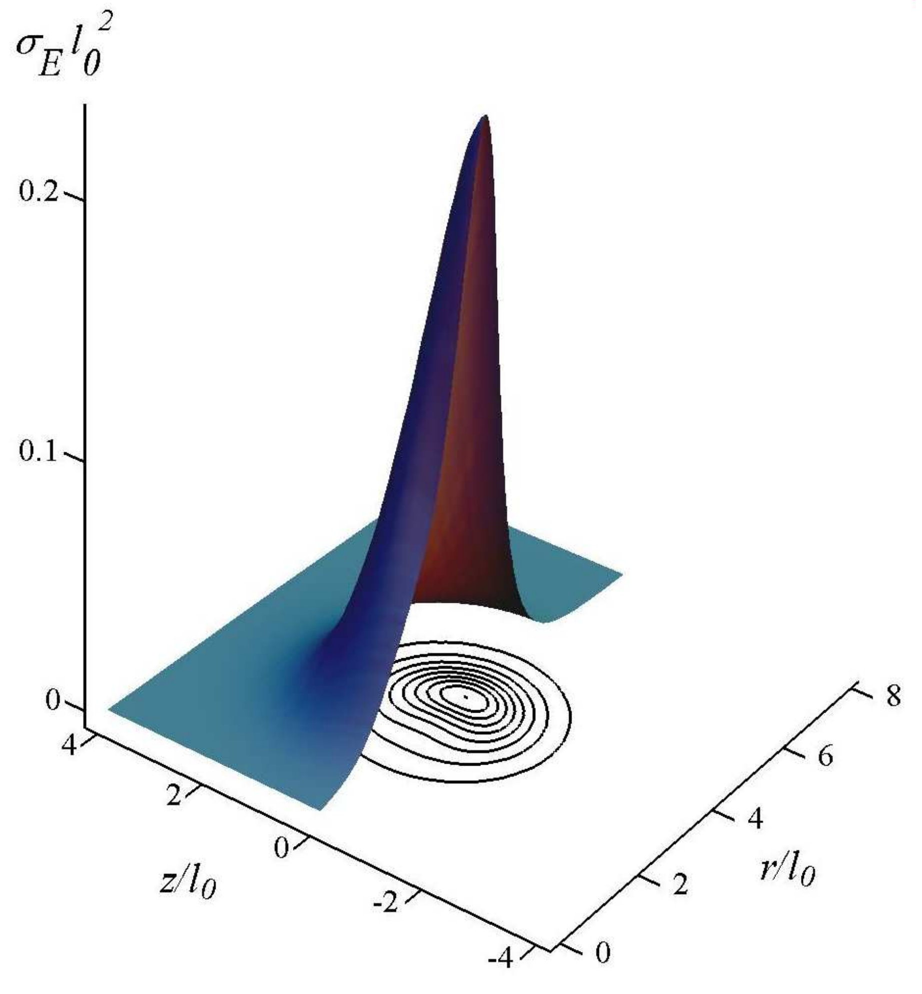
Energy distribution in hopfions for two energy branches

σ_E - normalized energy density in (r,z) plane

Lower energy branch



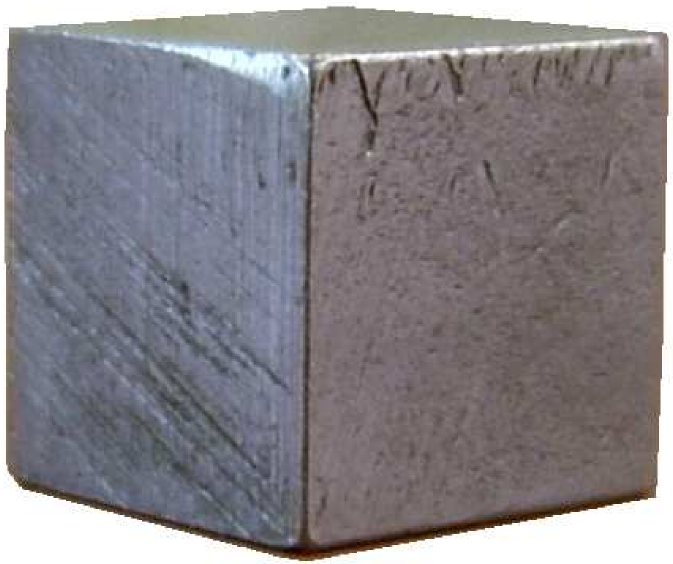
Higher energy branch



Localization area for hopfions ~ 20 l_0 in each dimension. For cobalt (^{27}Co), for example, l_0=5nm .

Metal cube: 10cm x 10cm x 10cm

3D grid of solitons inside this cube can contain 9.9 10^{17} hopfions.
If 1 hopfion equal to 1 bit of information, we have more than 10^5 Terabytes



VS

10^5 x