ULTRASHORT RELATIVISTICALLY STRONG SOLITONS IN PLASMA

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**Introduction**

**Initial 3d laser pulse**

- **nonlinear response factors:**
  - ✔ relativistic electron weighting
  - ✗ the push out of electrons from area occupied by laser pulse with ponderomotive force (transverse striction)
  - ✗ excitation of nonlinear oscillations of electrons behind laser pulse (wake field)
  - ✗ ions supposed to be motionless
nonlinear wave equation

dimensionless wave equation:

\[ 2 \frac{\partial^2 A}{\partial z \partial \tau} + \Delta_\perp A - \frac{A}{\sqrt{1 + |A|^2}} = 0 \]

\[ [\tau] = \omega_p^{-1} \]
\[ [x, y, z] = \omega_p^{-1} \cdot c \]
\[ [A] = A_0 = mc^2 / e \]

\( A \) - complex vector potential

the model consists of a single equation for complex value

\( \Re A = A_x, \quad \Im A = A_y \)

\( \tau \) - co-moving time:

\[ \tau = t - z \]

cylindrical symmetry:

\[ \Delta_\perp \Rightarrow \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right), \text{ where } r = \sqrt{x^2 + y^2} \]

initial conditions:

\[ A_x \sim \cos(\omega \tau), \quad A_y \sim \sin(\omega \tau) \]

\( \) circular polarization

no multiple spectrum generation
1. NSE research (smooth envelopes).

II “OPTICAL SOLITONS from fibers to photonic crystals” Y.S. Kivshar, G.P. Agrawal (2003)
III “SOLITONS nonlinear pulses and beams” N.N. Akhmediev and A. Ankiewicz (1997)

**existence and steadiness of 3d soliton structures was determined**

2. Switching to investigation of real field dynamics.

IV A.A. Balakin and V.A. Mironov, JETP Lett. 75 (12), 617 (2002)

**The common features of electromagnetic pulses dynamics were investigated in the case of Kerr type nonlinearity.**
limitations of the model

- **quasi-flat model:**
  \[
  \left(\frac{|A|}{A_0}\right)^3 \ll \frac{L_\|}{L_\perp}
  \]
  here \(A_0 = c^2 me^{-1}\) is the typical relativistic vector potential

- **non-refractive approach:**
  \[
  \omega_p \ll \omega
  \]

- **no wake field:**
  \[
  \frac{1}{\omega_p} \ll \frac{L_\|}{\omega c}
  \]
  wake field excitation is suppressed if the duration of laser pulse is much greater than plasma wave period

- thus the transverse push out of electrons from the area occupied by electromagnetic laser pulse could be neglected
Hamiltonian formalism

Integrals:

1. Energy integral:
   \[ H = \iiint\left(\frac{\partial A}{\partial r}\right)^2 + 2\sqrt{1 + |A|^2} - 2 \right) r \, dr \, d\tau \]

2. Quanta number integral:
   \[ I = \iiint\frac{\partial A}{\partial \tau} r \, dr \, d\tau \]

3. Impulse integral:
   \[ P = i\iiint\left(\frac{\partial A}{\partial \tau} A^* - \frac{\partial A^*}{\partial \tau} A\right) r \, dr \, d\tau \]

   (could not be obtained by standard routine for Hamiltonian systems)

Momenta evolution:

1. Centre of mass description:
   \[ \frac{d\tau_c}{dz} = -\frac{H}{I} \]

   Centre of mass velocity defined by initial conditions only.

2. Laser pulse width description:
   \[ \frac{d^2 r^2}{dz^2} = 8 \iiint \left(|A_r|^2 + \frac{-2 - |A|^2 + 2\sqrt{1 + |A|^2}}{\sqrt{1 + |A|^2}}\right) r \, dr \, d\tau \]

   Allow to investigate initial evolution of some types of solutions according to the set of it’s initial parameters.
**solitons**

**soliton general form:**

\[ A(\tau, z, r) = f(\xi, r)e^{i\varphi(\eta)} \]

where: \( f, \varphi \in \mathbb{R} \)

\[ \xi = \frac{\tau - Vz}{\sqrt{V}}, \quad \eta = \frac{\tau + Vz}{\sqrt{V}} \]

\( V \in (-\infty, +\infty) \) - is the free parameter

**solution:**

\[ \varphi(\eta) = a\eta \]

solution for phase \( a \in (-1, +1) \)

equation for amplitude

\[ \frac{1}{\rho^2} \frac{d}{dr} \left( \rho^2 \frac{df}{d\rho} \right) + a^2 f - \frac{f}{\sqrt{1 + f^2}} = 0 \]

Assumption of central symmetry for all 3 coordinates (two transverse spatial coordinates and co-moving time) allows to obtain quite simple differential equation for amplitude of soliton
two parameters:

- \( a \) - selects soliton amplitude, transverse width and spectral characteristics (number of optical periods in wave packet)

- \( V \) - scales whole soliton field along co-moving time coordinate
Lapunov’s method for partial derivative equations:

\[ F[A] = H + I - \lambda P \]

\[ \frac{\delta F[A]}{\delta A^*} = \begin{cases} 
- \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial f}{\partial r} \right) - \frac{\partial^2 f}{\partial \tau^2} + a^2 f + \frac{f}{\sqrt{1 + f^2}} + 2\lambda af = 0 \\
- \frac{\lambda}{\partial \tau} \frac{\partial f}{\partial \tau} - a \frac{\partial f}{\partial \tau} = 0 \Rightarrow \lambda = -a 
\end{cases} \]

Lapunov’s functional is combined of integrals of the equation under investigation thus it stays constant during the evolution.

\[ H + I > 8 \frac{P^2}{W} + 2 \frac{W}{1 + \sqrt{W}} \]

Here \( W = \int \! \int \! \int |A|^2 r \, dr \, d\tau \)

Numerical investigation of steadiness:

- Duration
  \[ w_t = (\int \! |\Psi|^4 (t - t_c) \, r \, dr \, dt) / W \]

- Transverse width
  \[ w_r = (\int \! |\Psi|^2 r^3 \, dr \, dt) / W \]
by varying initial delay $T_d$ between solitons, the relative phase before collision could be set.

$T_d = 52.9$

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results of two solitary waves interaction for different parameters of initial solitons.

surfaces of amplitude after interaction on axis of the system

\[ A(r = 0, \tau) \]

\( v1 = 1.47 \)
\( v2 = 1 \)
\( a = 0.995 \)

\( v1 = 1.9 \)
\( v2 = 1 \)
\( a = 0.995 \)

\( v1 = 2 \)
\( v2 = 1 \)
\( a = 0.995 \)

\( v1 = 1.7 \)
\( v2 = 1 \)
\( a = 0.995 \)

\( v1 = 1.8 \)
\( v2 = 1 \)
\( a = 0.995 \)

\( \varphi = 0.2\pi \)
longitudinal instability of a wave packet

initial gaussian pulse breaks apart on two solitary waves
solitons have different velocities
The break-up of non-perturbed super Gaussian wave packet allows to assume same processes in 3D case.
transverse instability of a flat wave (2D simulations)

The break-up of initial super gaussian pulse on a quantity of solitons in 2D model for nonsymmetrical perturbation nonliarity of 4th order allows to assume same processes in 3D case.

\[
\frac{\partial^2 A}{\partial z \partial \tau} + \frac{\partial^2 A}{\partial x^2} - \frac{A}{\sqrt{1+|A|^4}} = 0
\]

\[
|A|
\]

0 1 11 18 24 49 78 140

\[
\frac{\partial^2 A}{\partial z \partial \tau} + \frac{\partial^2 A}{\partial x^2} - \frac{A}{\sqrt{1+|A|^2}} = 0
\]

\[
|A|
\]

0 1 11 18 24 49 78 140

The break-up is slower in case of 2nd order of nonliarity.

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\[
\frac{\partial^2 A}{\partial z \partial \tau} + \frac{\partial^2 A}{\partial x^2} - \frac{A}{\sqrt{1+|A|^2}} = 0
\]

\[
|A|
\]
solitons in the full wave equation

For dense plasma the assumption of non refractive propagation is incorrect. In that case the full wave equation is appropriate:

\[
(1-v^2)\frac{\partial^2 A}{\partial \tau^2} + 2v \frac{\partial^2 A}{\partial z \partial \tau} - \frac{\partial^2 A}{\partial z^2} - \Delta_\perp A + \frac{A}{\sqrt{1+|A|^2}} = 0
\]

The last equation is obtained in new coordinates, moving with the velocity \(v\): \(\tau = t - v \cdot z, \quad z = z\)

Common form for solitary wave solution:

\[
A(\tau,z,r) = f(\tau,r)e^{i(\alpha \tau + bz)}
\]

The parameter \(b\) could be obtained with substitution of the solitary wave form to the wave equation:

\[
b = -\frac{(1-v^2)}{\alpha} \quad \Rightarrow \quad \frac{1}{\rho^2} \frac{\partial}{\partial \rho} \left( \rho^2 \frac{\partial f}{\partial \rho} \right) + \frac{1-v^2}{v^2} \alpha^2 f - \frac{f}{\sqrt{1+f^2}} = 0
\]

\[
\rho = \sqrt{\left(1-v^2\right)^{-1} \cdot \tau^2 + r^2}
\]

Thus the solutions of full wave equation could be obtained from ones demonstrated previously (with \(V=1\)) with simple transformations:

\[
\begin{array}{c|c|c}
\nu & \text{frequency} & \frac{1}{\sqrt{1-v^2}} \\
\sqrt{1-v^2} & \text{multiplication} & \text{longitudinal scaling for envelope}
\end{array}
\]
The mathematical approach for investigation of cold plasma and electromagnetic pulse interaction was developed. The generalization for the case of relativistic motions of plasma particles was performed.

The numerical code for the model describing nonlinear interaction of ultrashort laser pulse with plasma was developed (the code was optimized for distributed computation systems).

The solitons for the equation under investigation was obtained analytically as a generalization of NSE (Nonlinear Schrödinger equation).

The steadiness of found solitons was checked both analytically and numerically.

The detailed numerical analysis of solitons interaction was performed.

The processes of transverse and longitudinal instability of wave packets accompanied by formation of soliton type structures was demonstrated in numerical simulations.
thanks

find detailed information in article: