Experimental studies of capillary wave turbulence in dissipation region

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Capillary Turbulence

The dispersion relation of capillary waves:
\[ \omega^2 = \frac{\sigma}{\rho} k^3 \]

Three-wave interaction:
\[ \mathbf{k} = \mathbf{k}_1 + \mathbf{k}_2, \]
\[ \omega = \omega_1 + \omega_2 \]

Kolmogorov-Zakharov spectrum of capillary turbulence:
\[ \langle |\eta_\omega|^2 \rangle = C P^{1/2} (\sigma/\rho)^{1/6} \omega^{-17/6} \]

- External drive at low frequencies
- Energy transfer due to nonlinear interaction in the inertial interval
- Dissipation at high frequencies
Wave turbulence

- Weather predictions
- Technical applications
- Fundamental condensed matter and nonlinear physics

Turbulence of capillary waves on the surface of liquids

- Dynamics of waves on the sea surface
- Important for transfer and dissipation of energy in a high frequency domain
- Model studies and accurate check for predictions of the WT theory
### Properties of liquid hydrogen, helium and water

<table>
<thead>
<tr>
<th></th>
<th>Liquid Hydrogen, T=15K</th>
<th>Liquid Helium, T=4.2K</th>
<th>Water, T=300K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density, $\rho$, g/cm$^3$</td>
<td>0.076</td>
<td>0.145</td>
<td>1.0</td>
</tr>
<tr>
<td>Surface tension, $\sigma$, dyne/cm</td>
<td>2.7</td>
<td>0.12</td>
<td>77</td>
</tr>
<tr>
<td>Capillary length, $\lambda$, cm</td>
<td>0.19</td>
<td>0.030</td>
<td>0.28</td>
</tr>
<tr>
<td>Nonlinearity coefficient for capillary waves $V \sim (\sigma / \rho^3)^{1/4}$, cm$^{9/4}$/g$^{1/2}$ sec$^{1/2}$</td>
<td>8.9</td>
<td>2.5</td>
<td>3.0</td>
</tr>
<tr>
<td>Viscosity, $\nu$, cm$^2$/sec</td>
<td>0.0026</td>
<td>0.0002</td>
<td>0.01</td>
</tr>
<tr>
<td>Relative width of inertial range, $\omega_{\text{damping}} / \omega_{\text{drive}}$</td>
<td>60</td>
<td>65</td>
<td>20</td>
</tr>
</tbody>
</table>
Usage of liquid hydrogen as a test medium for studies of capillary turbulence

- High nonlinearity coefficient, low viscosity. The inertial range of frequencies is an order wider than in “conventional fluids” (water).

- Possibility to create quasi-2D charged layer below the liquid surface. The dispersion management by application of external electric field.

- Small density, excitation of surface oscillations by weak oscillating electrical field. Driving force acts directly on the surface.

- Spectral characteristic and angle dependence of the driving force can be varied in controllable way in wide limits.
Correlation function of the surface deviation

\[ <|\eta_\omega|^2| = <|\varphi_\omega/k|^2| \sim \omega^{4/3} <|\varphi_\omega|^2|, \]

\[ \varphi_\omega = k \eta_\omega \text{ — wave steepness.} \]

Instrumental function \( \Phi(\omega) \): \[ \varphi_\omega^2 \sim P_\omega^2/\Phi(\omega) \]

Narrow beam (\( ka << \pi \), \( a \) – size of laser spot):

\[ \Phi(\omega) \approx 1 \quad \rightarrow \quad <|\eta_\omega|^2| \sim \omega^{4/3} P_\omega^2 \]

Wide beam (\( ka >> \pi \)):

\[ \Phi(\omega) \sim \omega^{4/3} \quad \rightarrow \quad <|\eta_\omega|^2| \sim P_\omega^2 \]
Driving signal has been synthesised using the Fourier transform from a given amplitude spectrum and random phases. Digital-to-analog convector has been used to create driving random voltage.
Experimental results

Experimental spectra of surface oscillation $P_\omega^2$ in logarithmic scale. Waves have been excited by external random force in a frequency range 39—103 Hz. Straight segments of the spectra correspond to power-law distribution inside inertial interval.
Experimental spectra of surface oscillation $P_\omega^2$ in semi-logarithmic scale. The high frequency part of spectrum $P_\omega^2$ can be approximated well by exponential decay.
The decay exponent $\omega_d$ (blue points) as a function of external force power $W$ is fitted well by power-law $\omega_d \sim W^{0.4 \pm 0.1}$ (red line).
Experimental results

The boundary frequency for inertial range: \( \omega_b \approx 4.3 \text{ kHz} \)

The viscous dissipation begins from: \( \omega_d \approx 0.9 \text{ kHz} \)
The transition from Kolmogorov-Zakharov spectrum $<|\eta_\omega|^2> \sim \omega^{-17/6}$ to “quasi-Planck” distribution $<|\eta_\omega|^2> \sim \omega^b \exp(-\omega/\omega_d)$ for capillary turbulence has been observed for the first time. The experimental results support theoretical consideration for nonlinear waves in damping region [1] and are in qualitative agreement with numerical simulations [2].

The characteristic frequency $\omega_d$ of the “quasi-Planck” distribution grows with increase of the power $W$ injected into the wave system by external force. The functional dependence has been found $\omega_d \sim W^{0.4\pm0.1}$.