Experimental studies of capillary wave turbulence in dissipation region

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Capillary Turbulence

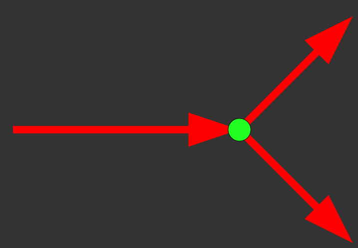
The dispersion relation
of capillary waves:

$$\omega^2 = \sigma/\rho\ k^3$$

Three-wave interaction:

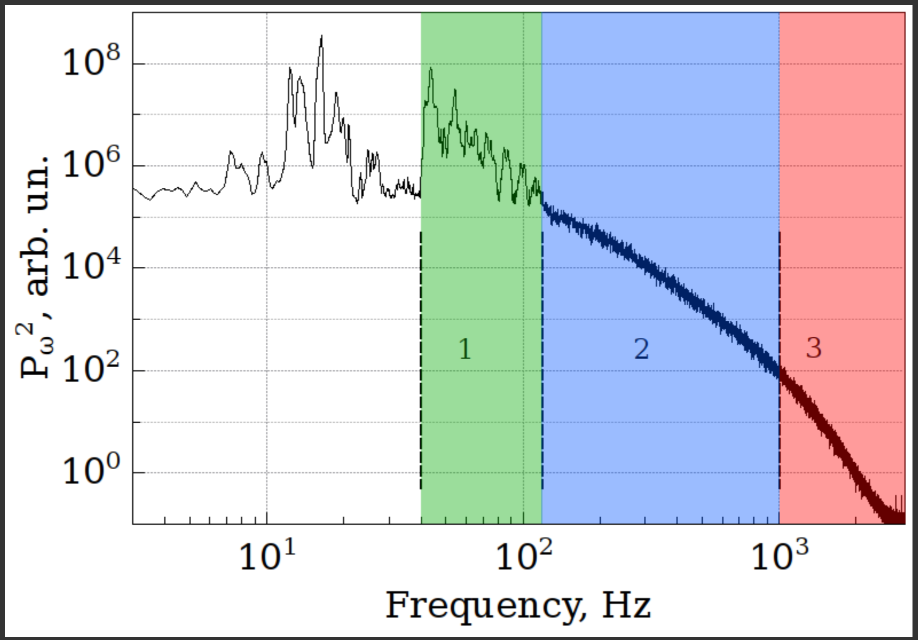
$$\boldsymbol{k} = \boldsymbol{k}_1 + \boldsymbol{k}_2,$$

$$\omega = \omega_1 + \omega_2$$



Kolmogorov-Zakharov spectrum
of capillary turbulence:

$$\langle|\eta_\omega|^2\rangle = C\ P^{1/2}\ (\sigma/\rho)^{1/6}\ \omega^{-17/6}$$



- External drive at low frequencies
- Energy transfer due to nonlinear interaction in the inertial interval
- Dissipation at high frequencies

Wave turbulence

- Weather predictions
- Technical applications
- Fundamental condensed matter and nonlinear physics

Turbulence of capillary waves on the surface of liquids

- Dynamics of waves on the sea surface
- Important for transfer and dissipation of energy in a high frequency domain
- Model studies and accurate check for predictions of the WT theory

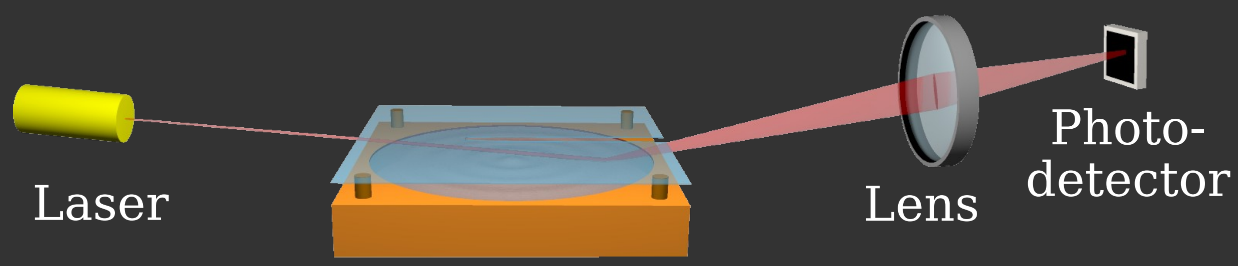
Properties of liquid hydrogen, helium and water

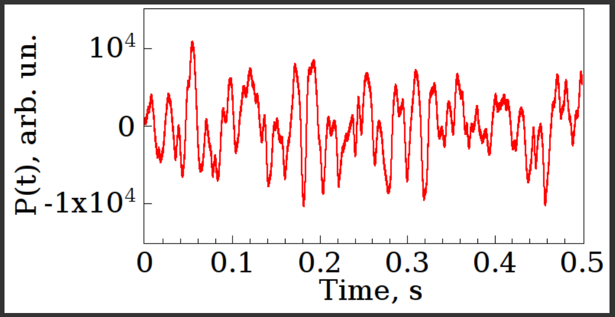
	Liquid Hydrogen, T=15K	Liquid Helium, T=4.2K	Water, T=300K
Density, ρ, g/cm^3	0,076	0,145	1,0
Surface tension, σ, dyne/cm	2,7	0,12	77
Capillary length, λ cm	0,19	0,030	0,28
Nonlinearity coefficient for capillary waves $V \sim (\sigma/\rho^3)^{1/4}$, cm$^{9/4}$/g$^{1/2}$ sec$^{1/2}$	8,9	2,5	3,0
Viscosity, υ, cm^2/sec	0,0026	0,0002	0,01
Relative width of inertial range, $\omega_{damping}/\omega_{drive}$	60	65	20

Usage of liquid hydrogen as a test medium for studies of capillary turbulence

- High nonlinearity coefficient, low viscosity. The inertial range of frequencies is an order wider than in “conventional fluids” (water).
- Possibility to create quasi-2D charged layer below the liquid surface. The dispersion management by application of external electric field.
- Small density, excitation of surface oscillations by weak oscillating electrical field. Driving force acts directly on the surface.
- Spectral characteristic and angle dependence of the driving force can be varied in controllable way in wide limits.

Experimental techniques





Example of recorded signal
(the power of reflected light $P(t)$).

Correlation function of the surface deviation

$$<|\eta_\omega|^2> = <|\varphi_\omega/k|^2> \sim \omega^{4/3} <|\varphi_\omega|^2>,$$

$\varphi_\omega = k\ \eta_\omega$ — wave steepness.

Instrumental function $\Phi(\omega)$: $\varphi_\omega^{\ 2} \sim P_\omega^{\ 2}/\Phi(\omega)$

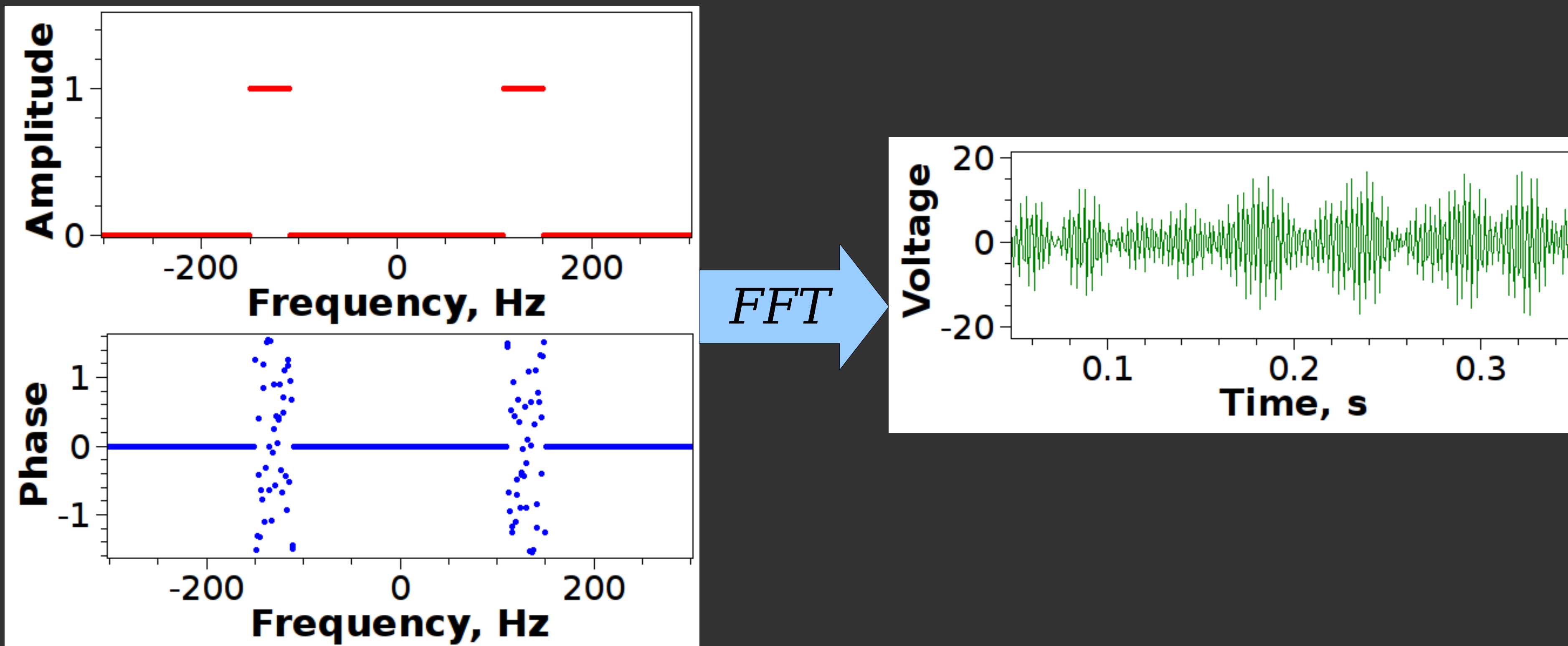
Narrow beam ($ka << \pi$, a – size of laser spot):

$\Phi(\omega) \simeq 1 \quad \rightarrow \quad <|\eta_\omega|^2> \sim \omega^{4/3} P_\omega^{\ 2}$

Wide beam ($ka >> \pi$):

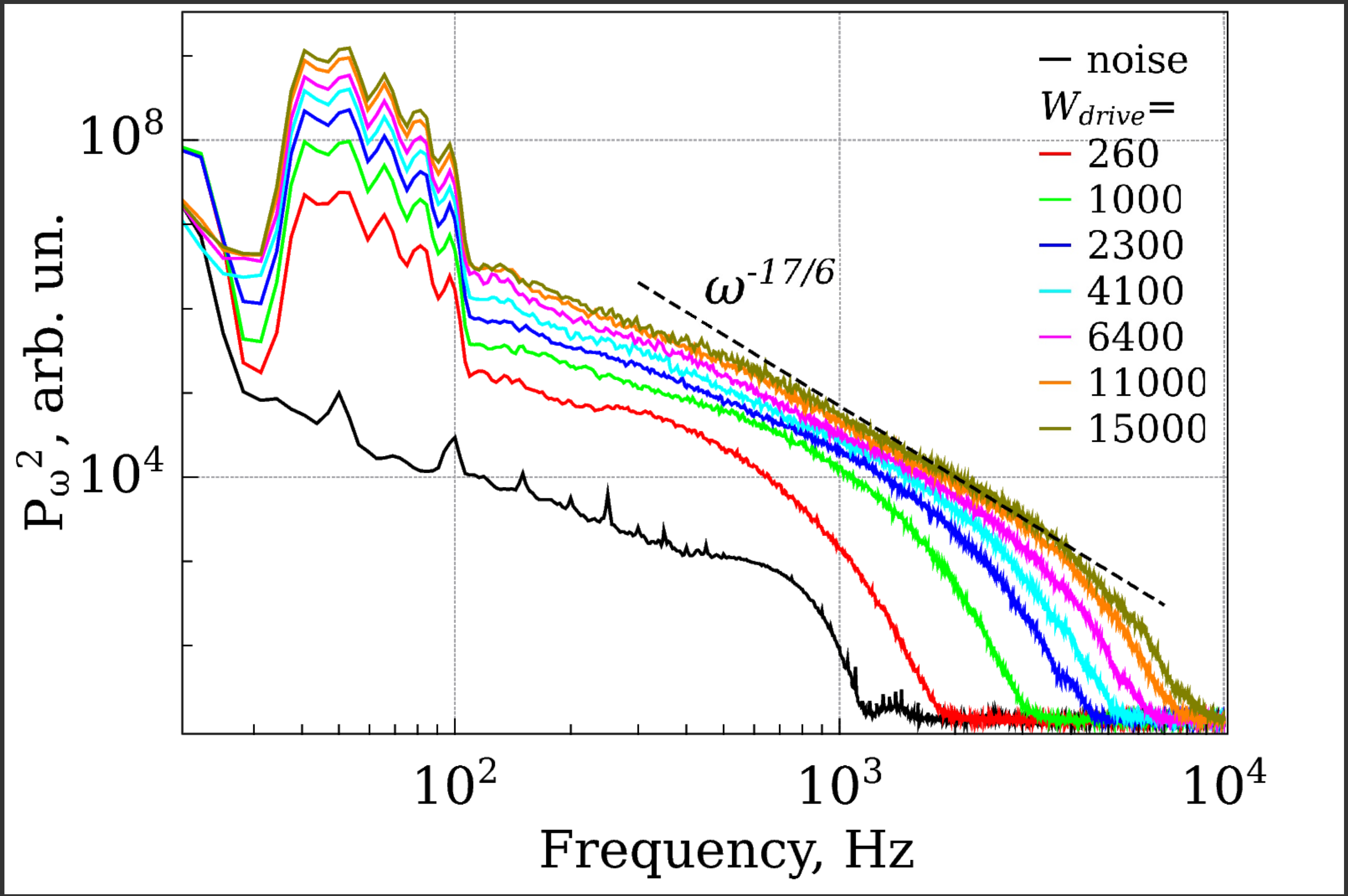
$\Phi(\omega) \sim \omega^{4/3} \quad \rightarrow \quad <|\eta_\omega|^2> \sim P_\omega^{\ 2}$

Experimental techniques



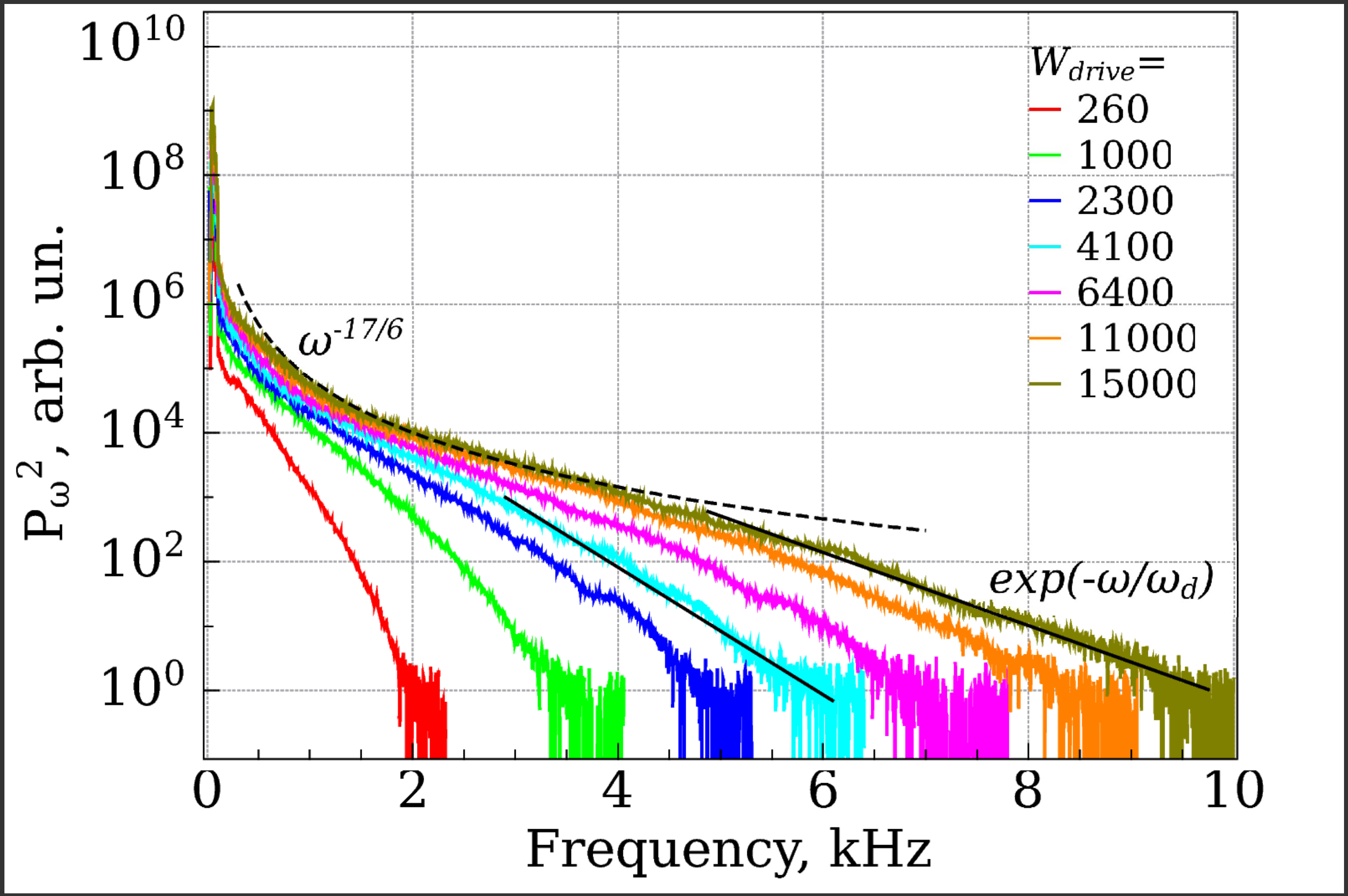
Driving signal has been synthesised using the Fourier transform from a given amplitude spectrum and random phases. Digital-to-analog convector has been used to create driving random voltage.

Experimental results



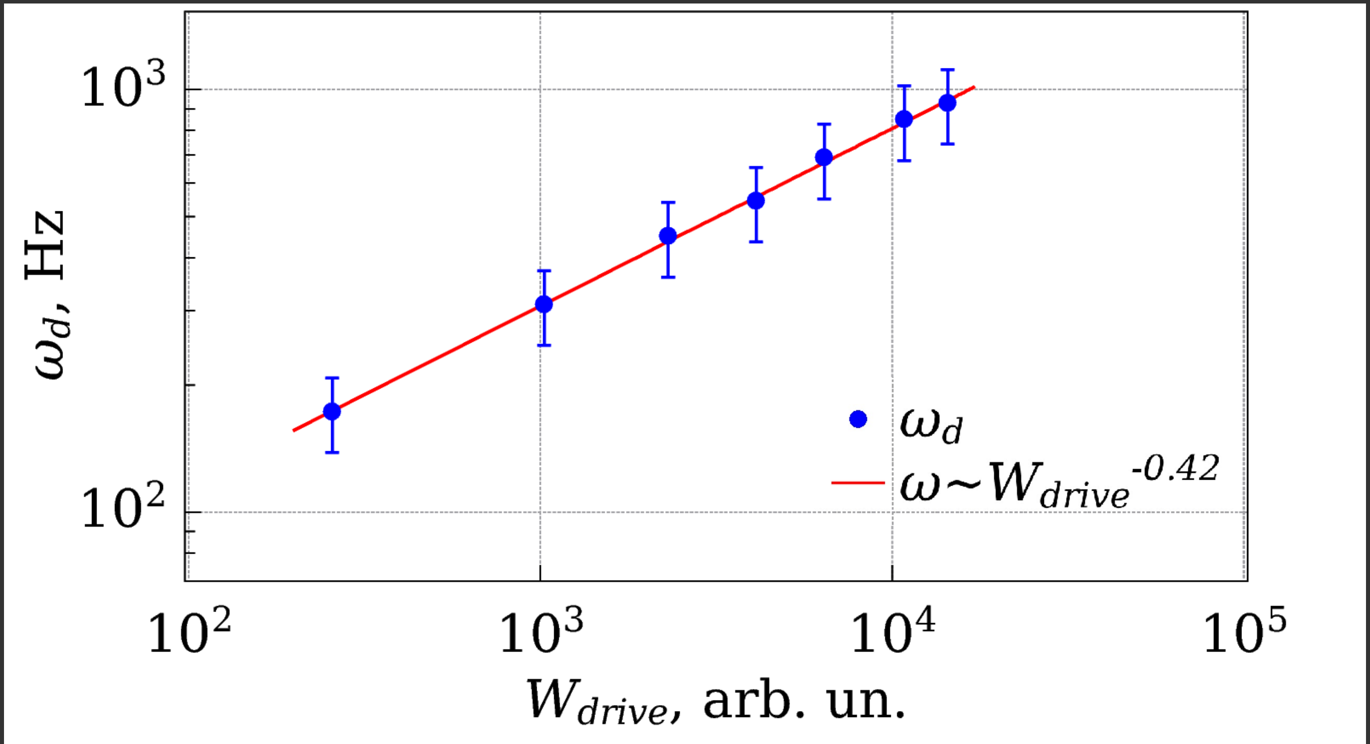
Experimental spectra of surface oscillation P_ω^2 in logarithmic scale. Waves have been excited by external random force in a frequency range 39—103 Hz. Straight segments of the spectra correspond to power-law distribution inside inertial interval.

Experimental results



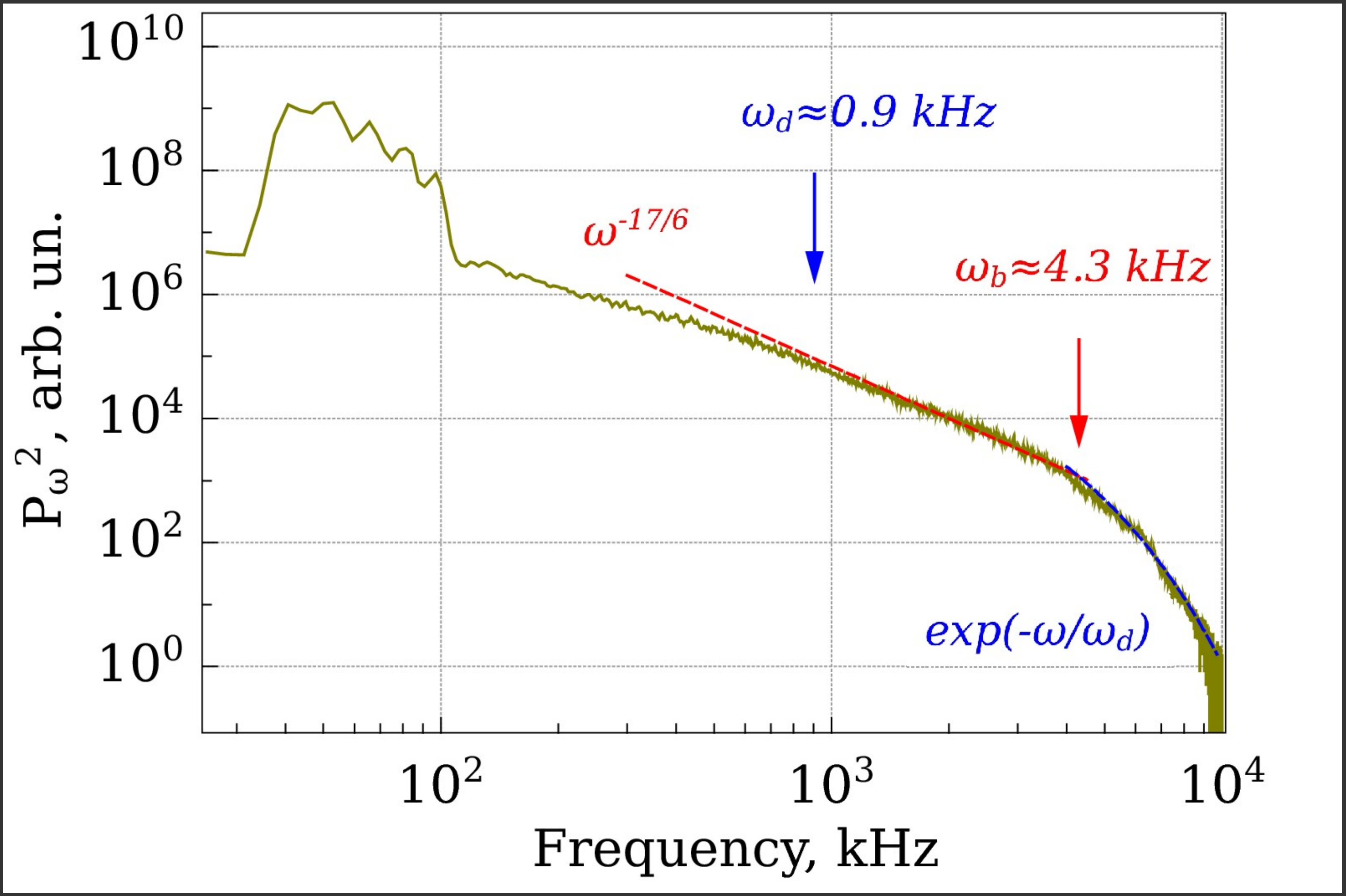
Experimental spectra of surface oscillation P_ω^2 in semi-logarithmic scale. The high frequency part of spectrum P_ω^2 can be approximated well by exponential decay.

Experimental results



The decay exponent ω_d (blue points) as a function of external force power W is fitted well by power-law $\omega_d \sim W^{0.4\pm01}$ (red line).

Experimental results



The boundary frequency for inertial range: $\omega_b \approx 4.3$ kHz

The viscous dissipation begins from: $\omega_d \approx 0.9$ kHz

Conclusions

The transition from Kolmogorov-Zakharov spectrum $<|\eta_\omega|^2> \sim \omega^{-17/6}$ to “quasi-Planck” distribution $<|\eta_\omega|^2> \sim \omega^b \, exp(-\omega/\omega_d)$ for capillary turbulence has been observed for the first time. The experimental results support theoretical consideration for nonlinear waves in damping region [1] and are in qualitative agreement with numerical simulations [2].

The characteristic frequency ω_d of the “quasi-Planck” distribution grows with increase of the power W injected into the wave system by external force. The functional dependence has been found $\omega_d \sim W^{0.4\pm01}$.

[1] V. M. Malkin, JETP, **86**, 1263 (1984)

[2] I. V. Ryzhenkova, G. E. Falkovich, JETP, **98**, 1931 (1990)