Quantized electromagnetic tornado in pulsar vacuum gap

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Giant pulses (GP), sporadically observed in a small number of pulsars Crab, B1937+21, B1821-24, B0540-69, B1112+50, B1957+20, J0218+4232, J1823-3021A, B0031-07 are a riddle which is not solved yet.

Giant pulses

Regular radiation

B1937+21
By A.Bilous

GP is characterized by enormous flux density (brightness temperature $\sim 10^{37}$ K), extremely small pulse duration (up to a few nanoseconds), circular polarization of both directions, power distribution by energies $P(E)$ and is mainly located in the narrow window with respect to average pulse position. These features fundamentally distinguish GPs from the ordinary pulses.
40 years to a pulsar problem

**Pulsars**, discovered in 1967 by **Jocelyn Bell** and **Anthony Hewish**, are rapidly spinning highly magnetized neutron stars whose light-house-like beams of radio waves sweep Earth, producing highly regular radio pulses.

\[ \Omega \sim \frac{1}{s}, \quad R_{NS} = 10\,\text{km} \]

\[ M_{NS} \sim M_{\text{sun}}, \]

\[ B_s \sim 10^{12}\,\text{G}, \]

\[ E \sim \Omega R B_s / c \sim 10^6\,\text{G}, \]

\[ \Gamma = \varepsilon / mc^2 \geq 10^7 \]
L. D. Landau was the first (1932) who has stated idea about existence in a nature of giant macroscopic atomic nuclei – neutron stars.

W. Baade and F. Zwicky is independent came to a conclusion about existence of neutron stars and connected them with super nova star explosions (1934).

A. Hewish has received the Nobel premium for detection pulsars on periodic radio radiation (1967).

Pulsars were identified with rotating neutron stars.
Periodic signals from space.
Antenna that was built by A.Hewish and his students for quick in time radio signals

By A.Hewish
40 years of pulsars
The first observation of pulses from pulsar (CP 1919)

First observation of pulses from CP 1919
28 November 1967

By A. Hewish
40 years of pulsars
The simplest beacon radiation model (Gold)

Magnetic field

\[ B \sim 10^{12} \, G \leq B_{\text{crit}} \]

\[ B_{\text{crit}} = \frac{me^2c^3}{eh} = 4.4 \cdot 10^{13} \, G \]

Plasma \( e^{\pm} \)

Rotating neutron star

\( P \sim 0.1 \to 1 \, S \)

\( (P_{\text{min}} = 0.00139 \, S) \)
The steadiness of the pulses makes pulsars very accurate clocks, rivaling the best atomic clocks on Earth.

**Lighthouse device:**

- fast rotation,
- strong magnetic field,
- strong electric fields,
- lightnings,
- acceleration of particles,
- creation an electron–positron plasma,
- Goldreich-Julian magnetosphere,
- polar inner vacuum gap

By A. Hewish

40 years of pulsars
Giant pulses at 4.9 GHz, Arecibo, 2002
**The task of the report**

**Giant pulses (GP)** record brightness temperature in Universe corresponding to the high energy density of $10^{15}$ erg/cm$^3$ looks as a key moment. Comparable densities of energy in the radio-frequency region are attainable in a cavity-resonator being the pulsar internal vacuum gap. Energy emission through the breaks accidentally appearing in the magnetosphere of open field lines corresponds to the GP. **Coulomb repulsion** of particles in the puncture spark in the gap leads to spark rotation around its axis in the crossed fields, which provokes the appearance of observed **circular polarization** of giant pulses. This rotation transforms the discharge into a some kind of the **electromagnetic tornado**. The quasi-classical and exact quantization of this rotation is possible.
Possible explanations of GP

Nevertheless, GP seems to be “a frequent, but rarely observed phenomenon inherent to all pulsars” (Soglasnov et al).

Offer previous explanations:
- strongly nonlinear effects in plasma
- modulation instability,
- Zakharov plasma wave collapse,
- reconnection of the magnetic field lines,
- induced scattering in narrow beams, etc.

Example of GP for B1112+50
A. A. Ershov, A.D. Kuzmin, 2003
Collapsing soliton prediction: Electric field (Weatherall, 1998)
Great radiation temperature of GP maybe the result of great energy density of radio oscillations in cavity – vacuum Polar gap.
Phase localization of GP

- The observed localization of the GP phase can be associated with radiation through the waveguides.

- In pulsar B1112+50 GPs are located in the center of the average pulse. We reckon that such localization may correspond to the radiation through the “waveguide” near the magnetic axis of the pulsar.

- If the GP phase corresponds to the “edge” of the average pulse then it is most likely to correspond with the radiation through the slot.

- The edge can be either retarded in comparison with the average profile (B1937+21), or advanced (J1823-3021A).

- That can correspond to rear or fore edges of the slots in the section of the telescope diagram.
Circular Polarization of GP

The observed impulse sequence is interpreted as a realization of a random electron acceleration (discharge) process in a vacuum gap over the polar cap (PC) – the open magnetic lines of force region on the surface of the neutron star. The gap may serve as a resonator-cavity with very high level of oscillation energy-density.

Stationarity of average pulse – the result of PC stationarity.

Goldreich-Julian charge density \( \rho_{GL} = -\left( \frac{B \Omega}{2\pi c} \right) \)
The Pulsar as fast particles generator

**Crab nebulae**
X-ray image from X-ray Space Observatory **CHANDRA**
0.1-10 KeV
September 2002

The **Crab Pulsar**
The relativistic **particles** spiral formed by rotating pulsar **magnetic field**

**Jet** from the Crab pulsar

The relativistic particles spiral formed by rotating pulsar magnetic field
The physical mechanism

- The physical mechanism giving such acceleration scheme may be a charge drain down from a sharp top of riplons in a strong parallel electric and magnetic field on a liquid neutron star Polar Cap surface heating by an inverse bombarding the Polar Cap and due to Ohm losses of discharge currents.
- Discharges may arise while the charges flow down from the microscopic edges of the polar cap surface, forming bunches and low frequency radiation that feeds the cavity.
- The edges may be destroyed in this process with later restoration.
Observed circular polarization is naturally explained by the peculiarities of the discharge in the vacuum gap.

Coulomb charge repulsion in the discharge bunch creates the radial electric field which is orthogonal to the magnetic one. Owing to the drift in the crossed fields it causes the discharge jet rotation around its axis and, accordingly, to the circular polarization of generated electromagnetic waves.

Owing to the drift the discharge channel turns into a peculiar vortex which resembles the well-known tornado.
Tornado in vacuum gap

\[ V = c \frac{[E, H]}{H^2} \]

\[ V_\varphi = \frac{cE}{H} \]

2D

\[ \varphi \propto \ln r \]
\[ E_r \propto \frac{1}{r} \]
\[ V_\varphi \propto \frac{1}{r} \]

\[ I = 2\pi r V_r \approx \text{const} \]

- Owing to the drift the discharge channel turns into a peculiar vortex with constant velocity circulation which resembles the well-known tornado.
- However, contrary to the hydrodynamic nature of the usual tornado, the vacuum gap tornado has a purely electromagnetic origin.
Coulomb field

\[ \text{div} \mathbf{E} = 4\pi \rho \]

\[ \rho(r) = \rho_0 \frac{r_0^2}{r_0^2 + r^2} \]

\[ E_r = \frac{2\pi \rho_0 r_0^2}{r} \ln \frac{r_0^2 + r^2}{r_0^2} \]

\[ \rho(r) = \rho_0 \frac{r_0^2}{r_0^4 + r^4} \]

\[ E_r = \frac{2\pi \rho_0 r_0^2}{r} \arctg \frac{r^2}{r_0^2} \]

- The Poisson equation for known 2D spatial charge gives us the radial to the vortex axis Coulomb field.
- Model radial distribution of the space charge: Unlocal Local
Dynamics of cylindrical bunch in local system.

Exact solution

Let us consider an **electromagnetic tornado** in the vacuum gap.

The **magnetic** field is uniform and directs along the discharge axis.

The **electric** space charge field is pure radial.

Equations in projections on the polar axes in x-y plane orthogonal to magnetic field are

\[
\frac{d\mathbf{p}}{dt} = e\mathbf{E} + \frac{e}{c}[\mathbf{V}, \mathbf{H}]
\]

\[\mathbf{p} = m\mathbf{V}\]

\[\mathbf{H} = (0, 0, H) \quad \mathbf{E} = E(r) \cdot \mathbf{e}_r\]

\[
\frac{de_r}{dt} = \frac{V_\varphi}{r} e_\varphi, \quad \frac{de_\varphi}{dt} = -\frac{V_\varphi}{r} e_r
\]

\[
\frac{dV_r}{dt} = \frac{V_\varphi^2}{r} + \frac{e}{m} \left( E + \frac{H \cdot V_\varphi}{c} \right)
\]

\[
\frac{dV_\varphi}{dt} = -\frac{V_r \cdot V_\varphi}{r} - \frac{eH}{mc} V_r
\]
The equations of motion in the plane orthogonal to a magnetic field in complex variables take the form

\[
\frac{d^2 \xi}{dt^2} - i\omega_c \frac{d\xi}{dt} - \frac{eE}{mr} \xi = 0
\]

\[
E = E(r)
\]

\[
\omega_c = eH / mc
\]

\[
\xi = \xi_0(r)e^{i\Omega(r)t}
\]

And take for angular velocity

\[
\Omega^2 - \omega_c \Omega + \frac{eE}{mr} = 0
\]
Rotation

Its roots $\Omega_{\pm}$ are:

$$2\Omega_{\pm} = \omega_c \pm \sqrt{\omega_c^2 - 4 \frac{eE}{mr}}$$

Electric field dependence of angular velocities

We have real roots for

$$\omega_c^2 > 4eE / mr$$
Pure **rotational** movement in the plane corresponds to requirement and leads to equation the solution of which is convenient to write through the angular velocities of rotation:

\[ V_r = 0 \quad \rightarrow \quad \frac{dV_\phi}{dt} = 0 \]

\[ \frac{dV_r}{dt} = 0 \]

\[ V_\phi^2 + \frac{eH}{mc} \cdot V_\phi + \frac{eE \cdot r}{m} = 0 \]

\[ V_\phi = \Omega(r) \cdot r \]

\[ \Omega_{\pm} = \frac{\omega_c}{2} \pm \sqrt{\left( \frac{\omega_c}{2} \right)^2 - \frac{eE}{m r}} \]

\[ \omega_c \equiv \frac{eH}{mc} \]

is the cyclotron frequency
Fast and slow rotation

In the pulsar conditions

Than the bigger root corresponds to cyclotron rotation. That movement is quantized, In the gap electrons are placed on zero landau level.

We are interested by the smaller root. It equals to

And describes the same drift movement that we consider.

\[ \omega_c \gg \frac{eE}{mr} \]

\[ \Omega_+ \approx \omega_c - \frac{eE}{2mr} \]

\[ \Omega_- \approx c \frac{E}{H} \cdot \frac{1}{r} \]

\[ V_\phi = c \frac{E}{H} \]
Possible quantization of vortex

The quasi-classical quantization is possible by using the condition, which leads to the angular frequency and to the macroscopic particle linear density on the axis that does not contain any model parameters and only depends on the magnetic field.

\[ mrV_\varphi = (n + \frac{1}{2})\hbar \]

\[ \Omega_n = \frac{(n + \frac{1}{2})\hbar}{2mr^2} \]

Rotation frequencies form the bands the borders of which are determined by tornado internal and external radii.

\[ r \approx 10^{-5} \implies \Omega \approx 10^{10} \]

Linear particle density:

\[ N \approx \frac{\hbar B}{emc}(n + \frac{1}{2}) \]

Current in discharge:

\[ I = ecN \approx \frac{\hbar B}{m}\left(n + \frac{1}{2}\right) \]

Such structure might explain the frequency stripes observed in GP spectrum.

The full Goldreigh-Julian current flowing through the gap is provided by \( q \) vortex lines. In the ground state it is required for that about \( 10^7 \) lines. For a smaller \( q \) the Rydberg states with \( n>>1 \) is required.
Main pulse

Why?

Interpulse

Strips in spectrum
Berry phase for e-m tornado

After K.Yu.Bliokh and author (B&K)

Spin precession – equation BMT (Bargmann, Michel & Telegdi)

\[ \frac{d \vec{S}}{dt} = \vec{S} \times \left[ \frac{e c \vec{B}}{\varepsilon_0} - \frac{e c^2}{\varepsilon_0 (\varepsilon_0 + mc^2)} \vec{p} \times \vec{E} \right] \]

The quantization rule is

\[ \hbar^{-1} \oint \varepsilon dt + \theta_B = 2\pi n \]

\[ \theta_B \] is a Berry phase

In the ultra relativistic case we have due to contribution of the Berry phase

\[ \frac{R \varepsilon_0}{V \hbar} = n + S_z = n \pm \frac{1}{2} \]

\[ p \times \vec{E} \parallel \vec{B} \parallel \vec{Oz} \Rightarrow S_z = \text{const} \]

\[ \varepsilon_0 = \sqrt{m^2 c^4 + p^2 c^2} \]

\[ S_z = \pm \frac{1}{2} \] – the spin eigenvalues.
Viet, Coulomb and Lorentz

Let us discuss what give us for \( V_r \) the eq. (***) in the case \( \Omega = \Omega_+ + \Omega_- \)

Rewrite it in the form

\[
\frac{dV_r}{dt} = (\Omega - \Omega_+)(\Omega - \Omega_-) \cdot r
\]

After substitution we have

\[
\frac{dV_r}{dt} = \Omega_+ \Omega_- \cdot r
\]

Due to Viet theorem

\[
\Omega_+ \Omega_- = \frac{eE}{mr}
\]

And for \( \frac{dV_r}{dt} \) we have in this case

\[
\frac{dV_r}{dt} = \frac{eE}{m}
\]

(as for \( H = 0 \))

Taking into account (one more due to Viet theorem) \( \Omega_+ + \Omega_- = \omega_c \) we have

\[
\begin{align*}
\frac{dV_r}{dt} - \frac{V^2}{r} + \omega_c V_\varphi - \frac{eE}{m} &= 0, \\
\frac{dV_\varphi}{dt} + \frac{V_\varphi V_r}{r} - \omega_c V_r &= 0.
\end{align*}
\]

that stationary rotation with angular velocity \( \omega_c \)

will be accompany by radial expansion

while stationary rotation with angular velocity \( \Omega_+ \)

takes place in condition of exact compensation of radial forces

(\textbf{Coulomb} and \textbf{Lorentz}).
• Relativistic generalizations, as easy may be seen achieve by change of quantities in equations
where -
Is the resulting Lorentz-factor of movement along the magnetic field and rotational movement.

• The real solutions need in

\[ m \rightarrow m \cdot \Gamma \]

\[ \Gamma = \Gamma_1 \cdot \Gamma_2 \]

\[ \left( \frac{\omega_c}{2} \right)^2 \geq \frac{eE}{mr} \]

On the figure.
The dependence of angular velocities from the radial electric field
Stability conditions on the tornado axes

\[ \frac{E}{r} \rightarrow 2\pi \rho_0, \quad r \rightarrow 0 \]

\[ \rho_0 \]
- Charge density on the tornado axes.

\[ \omega_c^2 \geq 2\omega_p^2 \]

\[ \frac{H^2}{8\pi} \geq n_0 \cdot mc^2 \]
Circular polarization

The solution

\[ V_\varphi = \Omega_\pm(r) \cdot r \]

describes the electromagnetic tornado in the vacuum gap where the repulsion field of the space charge is compensated by the Lorentz force, the radial movement is absent, and rotation is specified by the drift in the crossed fields.

Due to this rotation, the circular polarization will appear in the discharge radiation. Really, inescapable small deviations of the charge density from the ideal axial symmetry will bring to appearance of the electric field rotated around the bunch axis and connected with it the circular polarization of radiation.
The discussed movement is stable at small disturbances:

\[ \omega_c \gg \frac{eE}{mr} \]

\[ V_\phi \gg V_r \neq 0 \]

\[ \frac{dV_\phi}{dt} = -V_r \left( \frac{V_\phi}{r} - \omega_c \right) \]

\[ \frac{dV_r}{dt} \approx \left( \frac{2V_\phi}{r} - \omega_c \right) \frac{\delta V_\phi}{\delta t} \delta t \approx -V_r \cdot \omega_c^2 \delta t \]
Instability of metal surface in strong orthogonal electric field

- Tonks, Phys.Rev. 1935
- Frenkel, JETP, 1936
- Kuznetsov & Spector,
- Tip sharpness and droplet detachment

N.M. Zubarev, JETP, 116, 1999
В излучении гигантских импульсов обнаруживаются оба знака поляризации. При отражении импульса от стенок резонатора знак поляризации может изменяться. Однако, если речь идет об излучении во время импульса индивидуального разряда, этого объяснения недостаточно. Можно предположить, что неустойчивость в сильном электрическом поле приводит к выбросу плазмы, содержащей заряды обоих знаков – как электроны, так и ионы. Сгусток поляризуется так, что на одном его конце возникает избыток зарядов одного, а на втором - другого знака. Поэтому его концы приходят во вращение в противоположных направлениях, что может обеспечить появление в излучении поляризаций обоих знаков. Заметим, что параметры дрейфа определяются только полями и не зависят от масс дрейфующих частиц.
Interpuls:

Puls:

Power-law spectrum

Bilous A.V.
АКЦ ФИАН, ФАКИ МФТИ
For the particle **drift** velocity at large distances from the axis we have a movement with **constant circulation** and solid-type rotation on the small distances. (The latter corresponds the finite charge density on the vortex axis.)

Thus, owing to the drift the discharge channel turns into a peculiar vortex which resembles the well-known tornado.

However, contrary to the hydrodynamic nature of the usual tornado, the vacuum gap tornado has a purely electromagnetic origin.
Quantum tornado

Let us discuss a non relativistic tornado in the pulsar vacuum gap. In the states with stationary rotation without of radial movement which we are interested the energy is conserved and we can consider the stationary Schrodinger equation, where Hamiltonian is

$$\hat{H} = \frac{1}{2m} \left( \mathbf{p} - \frac{e}{c} \mathbf{A} \right)^2 + \hbar \omega_c \cdot \sigma_z + e\phi(r)$$

Momentum operator  Vector-potential  spin  Space charge Electric potential

$$\mathbf{p} = \frac{\hbar}{i} \nabla$$  $$\mathbf{A} = \frac{1}{2} \mathbf{H} \times \mathbf{r}$$  $$\sigma_z = \pm \frac{1}{2}$$  $$\phi(r)$$
Tornado Hamiltonian in cylinder coordinates

In magnetic field \( \mathbf{H} = H \mathbf{e}_z \)

we have for vector potential \( A_z = A_r = 0, \ A_\phi = Hr / 2 \)

That differs from the known Landau potential but is preferable due to dependence electric field potential on \( r \). Momentum components

\[
I_\phi = \frac{\hbar}{i} \frac{\partial}{\partial \phi}, \quad p_z = \frac{\hbar}{i} \frac{\partial}{\partial z}
\]

are commutate with the Hamiltonian and have joint with it eigenvalues but \( p_r = \frac{\hbar}{i} \frac{\partial}{\partial r} \)

is not commutate due to dependence of ors of axes \( r \) and \( \phi \) on polar angel.

Due to that we have

\[
\hat{H} = -\frac{\hbar^2}{2m} \left( \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} + \left( \frac{\partial}{r \partial \phi} - \frac{i e Hr}{2 \hbar c} \right)^2 \right) + \hbar \omega_c \cdot \sigma \ + e\phi(r)
\]
Rotation frequencies
(perspectives)

Wave function \( \psi = \chi(r) \exp\left\{ \frac{i}{\hbar} p_z z \right\} \exp\left\{ \frac{i}{\hbar} mr^2 \Omega \Phi \right\} S(\sigma_z) \)

\[
\hat{H} = -\frac{\hbar^2}{2m} \left( \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \right) + \frac{mr^2}{2} \left( \Omega - \omega_L \right)^2 + \hbar \omega_c \cdot \sigma_z + e\phi(r) \quad \omega_L \equiv \frac{\omega_c}{2}
\]

For \( \Omega = \Omega_\pm \) where \( \Omega_\pm = \omega_L \pm \sqrt{\omega_L^2 - eE / mr} \) we have

\[
\hat{H} = -\frac{\hbar^2}{2m} \left( \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \right) + \frac{mr^2}{2} \left( \pm \sqrt{\omega_L^2 - eE / mr} \right)^2 + \hbar \omega_c \cdot \sigma_z + e\phi(r) \quad \text{or}
\]

\[
\hat{H} = -\frac{\hbar^2}{2m} \left( \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \right) + \frac{mr^2 \omega_L^2}{2} + \hbar \omega_c \cdot \sigma_z - \frac{eEr}{2} + e\phi(r)
\]
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Decametric wave radio telescope
UTR-2 of RI NANU, Kharkov
With the best wishes!