Turbulent Quasi-Breathers in MMT model

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MMT model

$$i\frac{\partial \Psi}{\partial t} = \left|\frac{\partial}{\partial x}\right|^{\alpha} \Psi + \lambda \left|\frac{\partial}{\partial x}\right|^{\beta/4} \left[\left| \left|\frac{\partial}{\partial x}\right|^{\beta/4} \Psi \right|^{2} \left|\frac{\partial}{\partial x}\right|^{\beta/4} \Psi \right], \quad \lambda = \pm 1$$

$$N = \int |\Psi|^2$$

$$H = \int \left| \left|\frac{\partial}{\partial x}\right|^{\alpha/2} \Psi \right|^2 dx + \frac{1}{2}\lambda \int \left| \left|\frac{\partial}{\partial x} \Psi\right|^{\beta/4} \right|^4 d\,x$$

$$M = \frac{1}{2}\int \left(\Psi \frac{\partial \Psi^*}{\partial x} - \frac{\partial \Psi}{\partial x}\Psi^* \right) dx$$

Majda, McLaughlin, Tabak 1997

Zakharov, Kuznetsov 1998

Zakharov, Guyenne, Dias, Pushkarev, 2001, 2004

Rumpf, Newell, Zakharov 2009

Deep Water Case:

$$i\frac{\partial \Psi_k}{\partial t} = |k|^{\alpha}\Psi_k + \int T_{k k_1 k_2 k_3} \Psi^*_{k_1} \Psi_{k_2} \Psi_{k_3} \delta_{k+k_1-k_2-k_3} dk_1\, dk_2\, dk_3$$

$$T_{k k_1 k_2 k_3} = \left|k^{\beta/4}\right|\left|k_1{}^{\beta/4}\right|\left|k_2{}^{\beta/4}\right|\left|k_3{}^{\beta/4}\right|$$

$$\eta(x,t) = \int_{-\infty}^{+\infty} e^{ikx} \sqrt{\frac{\omega_k}{2}} \left(\hat{\Psi}_k + \hat{\Psi}^*_{-k}\right) dk$$

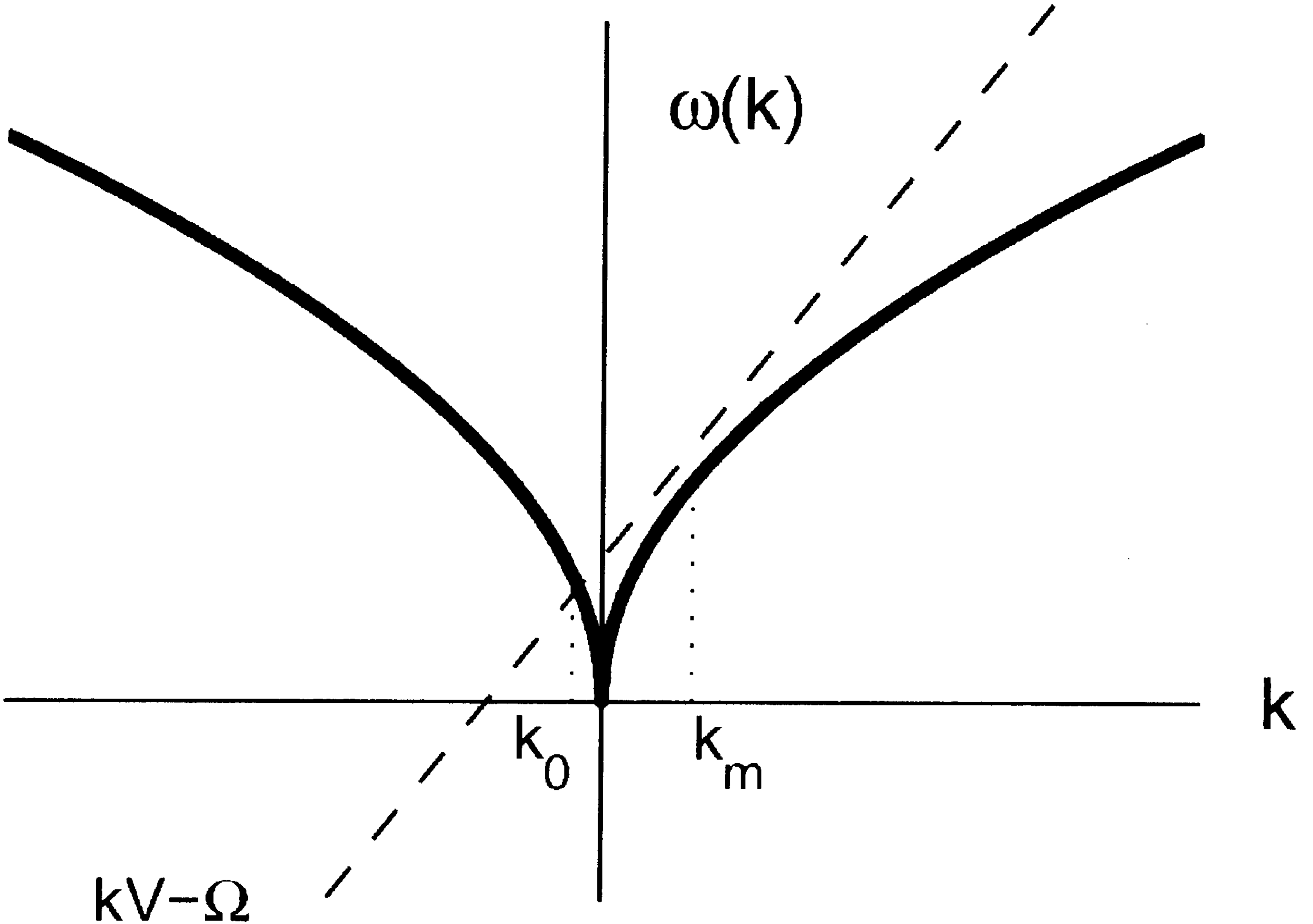
$$\lambda = +1, \quad \alpha = \frac{1}{2}, \quad \beta = \frac{3}{4}$$

Quasi-Solitons

Kuznetsov, Zakharov 1998

$$\Psi_k(t) = e^{i(\Omega - kV)t}\,\phi_k$$

$$\phi_k = -\frac{\lambda}{\Omega - kV + \omega_k}\int (k\,k_1\,k_2\,k_3)^{\beta/4}\,\phi_1\,\phi_2\,\phi_3^*\,\delta(k_1 + k_2 - k_3 - k_4)\,dk_1\,dk_2\,dk_3$$


ω(k)
k
k0
km
kV−Ω

Approximate solution in the form of NLS soliton:

$$\phi(x) = \sqrt{\frac{\alpha(1-\alpha)}{k_m^{\beta-\alpha+2}}} \frac{q}{\cosh qx}, \qquad \frac{q}{k_{max}} \ll 1$$

$$\Psi(x,t) = \phi(x - vt)\, e^{i\Omega t} e^{i k_m (x - vt)}$$

$$\Omega = -(1-\alpha) k_m^{\alpha} - \frac{1}{2}\alpha(1-\alpha) k_m^{\alpha-2} q^2$$

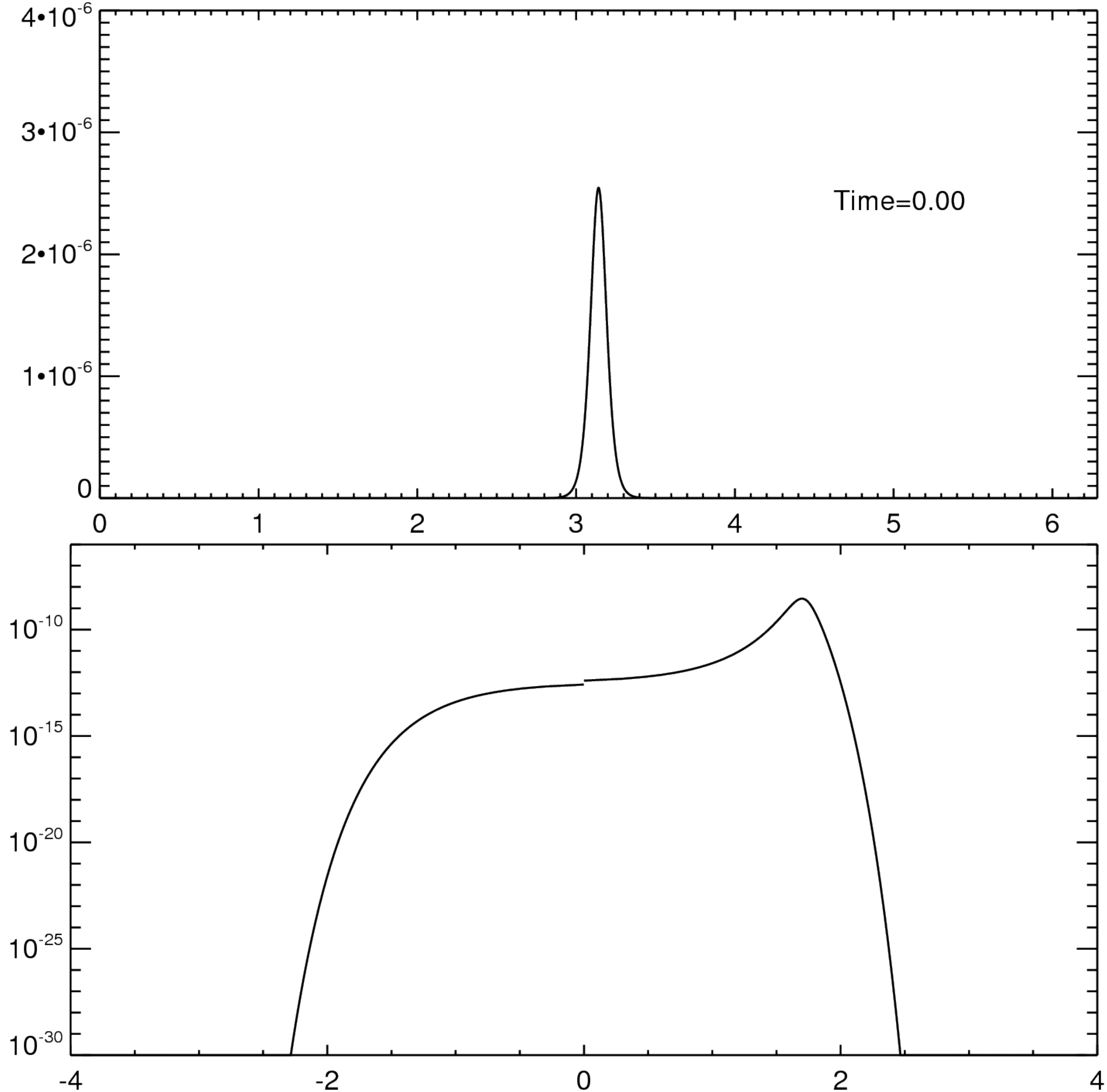
$$V = \alpha k_m^{\alpha-1}$$

Self-similar solution:

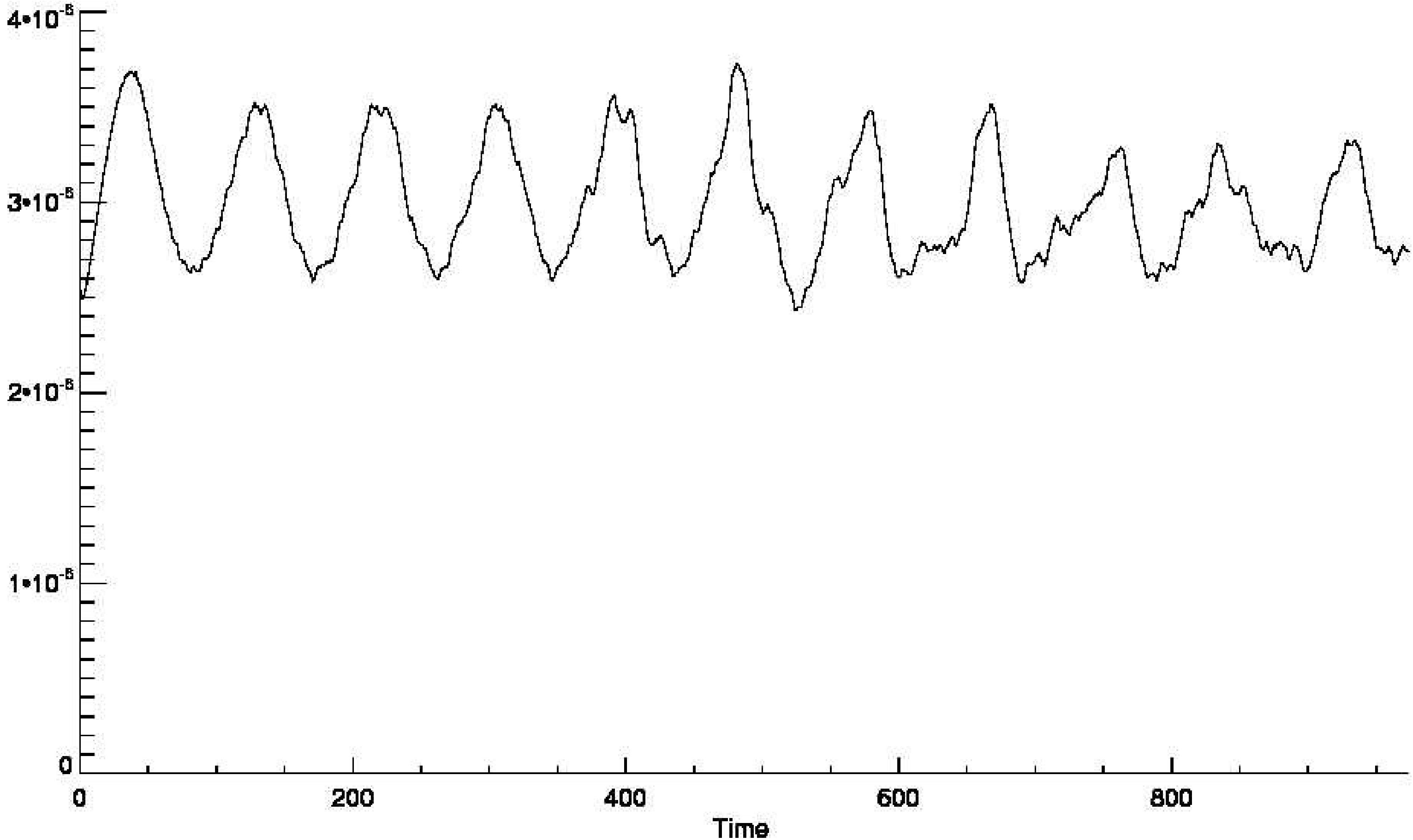
$$\Psi(k,t) = (t_0 - t)^{q+i\nu}\,\phi\left((t_0 - t)^{1/\alpha} k\right)$$

$$q = \frac{1}{2\alpha}(\beta + 2) - \frac{1}{2}$$

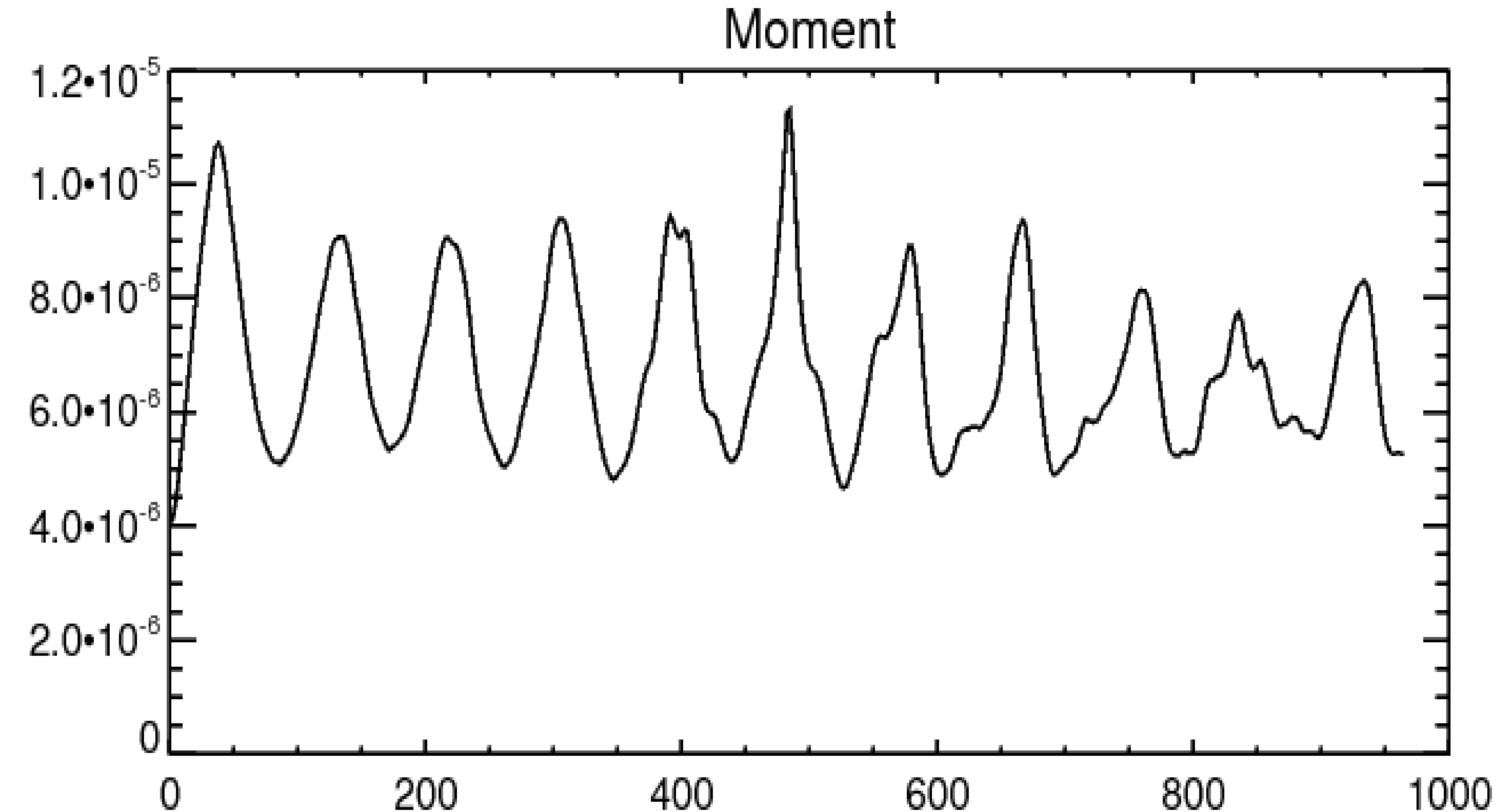
$$|\psi|^2 \sim k^{-9/2}$$



Time=0.00
4•10-6
3•10-6
2•10-6
1•10-6
0
0 1 2 3 4 5 6
10-10
10-15
10-20
10-25
10-30
-4 -2 0 2 4

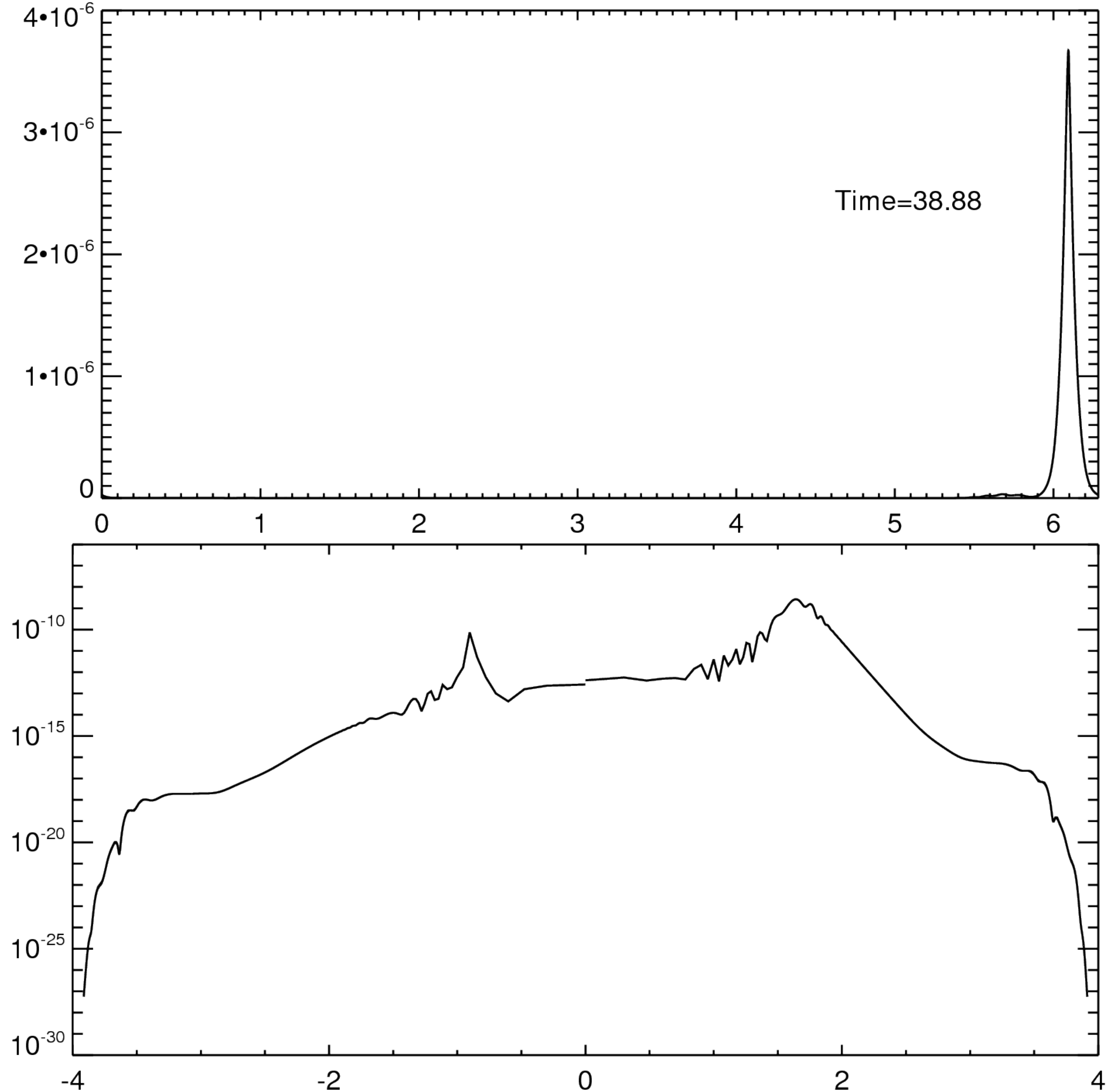


Real-space maximum as a function of time

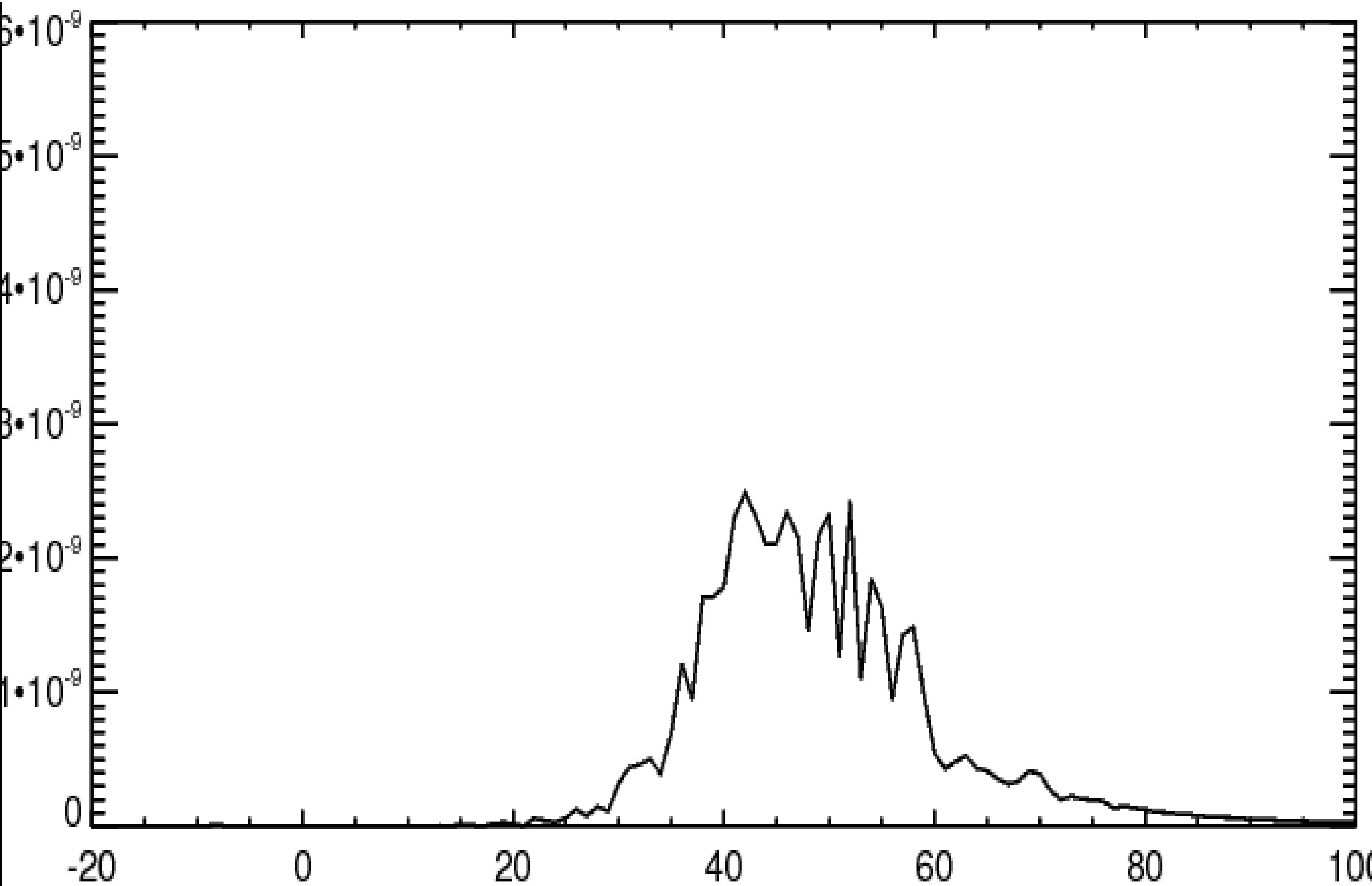


$$N = \int (k - k_0)^2 |\Psi|^2 \, dk$$

$$k_0 = \frac{\int k |\Psi|^2 \, dk}{\int |\Psi|^2 \, dk}$$



4•10^{-6}
3•10^{-6}
2•10^{-6}
1•10^{-6}
0
Time=38.88
0
1
2
3
4
5
6
10^{-10}
10^{-15}
10^{-20}
10^{-25}
10^{-30}
-4
-2
0
2
4


6•10-9
5•10-9
4•10-9
3•10-9
2•10-9
1•10-9
0
-20
0
20
40
60
80
100