FREAK WAVES AND CONCLUSIVE SIMULATION

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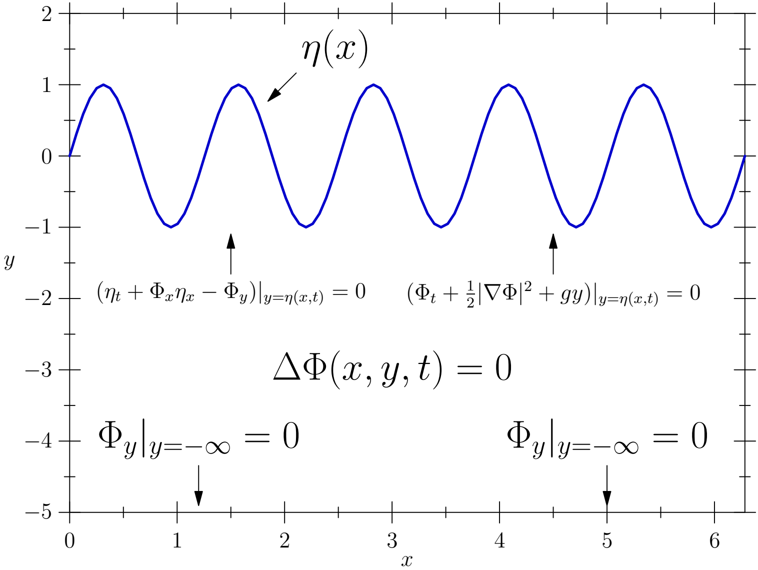
Mathematical model



Conformal variables

Introduce the following pair of complex functions:

$$z(w,t) = x(w,t) + iy(w,t)$$

and

$$\Pi(u,t) = \Psi(u,t) + i\mathrm{H}[\Psi(u,t)],$$

where $w = u + iv$. Introduce new variables $R(w,t)$ and $V(w,t)$ as follows:

$$R(w,t) = \frac{1}{z_w} \quad \text{and} \quad V(w,t) = i\frac{\Pi_w}{z_w}.$$

The functions R and V are analytic on the lower half-plane and the following conditions are fulfilled:

$$R(w,t) \to 1, \quad |w| \to \infty, \quad \mathrm{Im}\, w \leqslant 0,$$
$$V(w,t) \to 0, \quad |w| \to \infty, \quad \mathrm{Im}\, w \leqslant 0.$$

Dyachenko equations

R and V satisfy the following system of integro-differential equations:

$$R_t = i(UR_w - U_w R), \tag{1}$$

$$V_t = i(UV_w - B_w R) + g(R - 1), \tag{2}$$

where

$$U = \mathrm{P}(V\overline{R} + \overline{V}R),\ B = \mathrm{P}(V\overline{V}),\ \mathrm{P} = \frac{1}{2}(\mathrm{I} + i\mathrm{H})\,. \tag{3}$$

Such equations are called Dyachenko equations.

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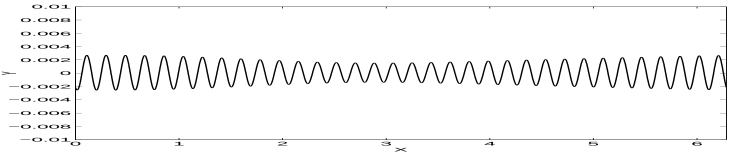

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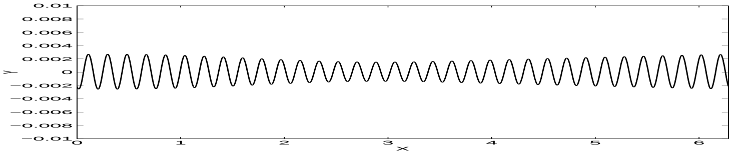

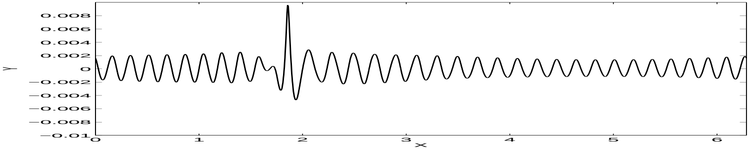

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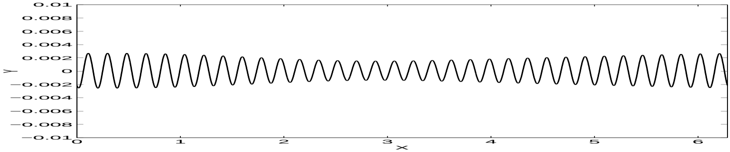

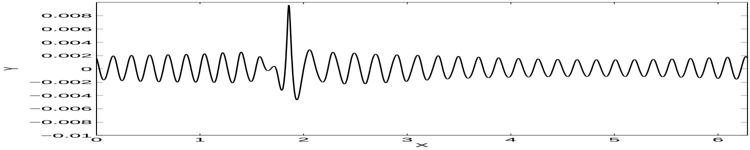

But! Where a guarantee what is a freak wave, instead of result of an numerical error?

Estimation of lifetime of solutions

For $s > 0$ denote unbounded domain
$Q_s = \{w = u + iv \in \mathbb{C} : 0 < u < 2\pi, -\infty < v < s\}$.
Introduce functional spaces E_s. This spaces are consist from analytic function in Q_s:

$$f = \sum_{k=0}^{\infty} f_k e^{-ikw} \quad \text{with norm} \quad \|f\|_{E_s}^2 = \sum_{k=0}^{\infty} |f_k|^2 e^{2sk}.$$

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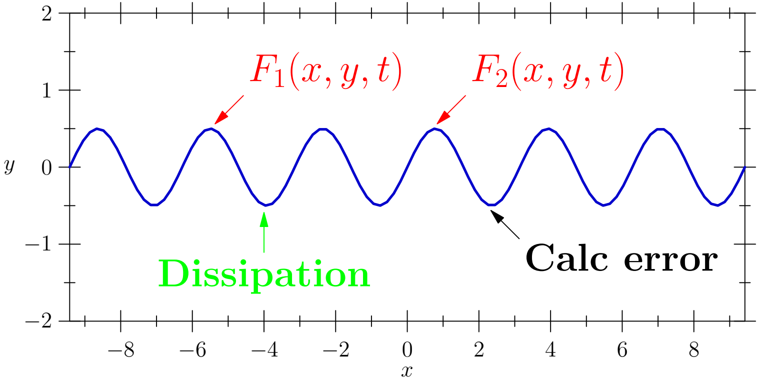
Theorem

Let R_0, $V_0 \in E_s$, then there exists solution $R, V \in E_{s'}$ $(0 < s' < s)$ on $[0, T(s', R_0, V_0)]$. In addition function $T = T(s', R_0, V_0)$ is recursive function.

«Useful» errors!



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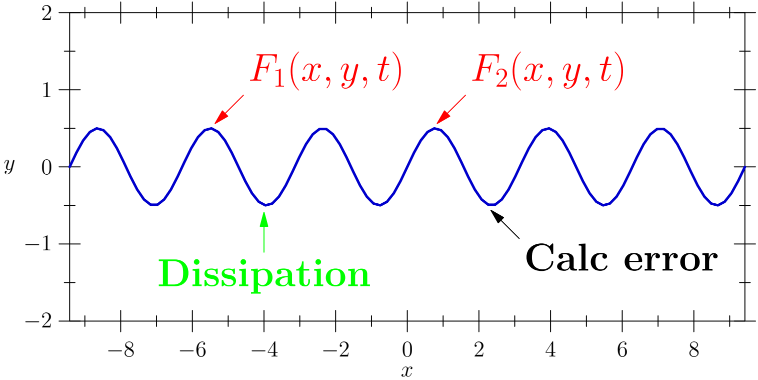

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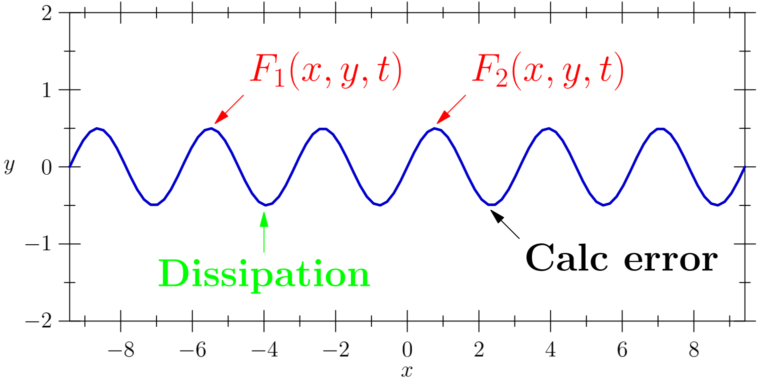

Errors of calculations ⇒ Small external influences (wind)!
Errors of calculations provide stability of our solutions. In particular, by us it is shown that freak waves can arise and at small external influences.

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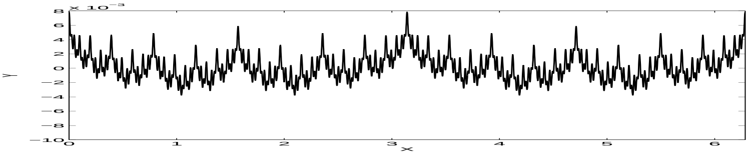

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Thank you for attention!