Mirror structures in magnetized plasma with pressure anisotropy

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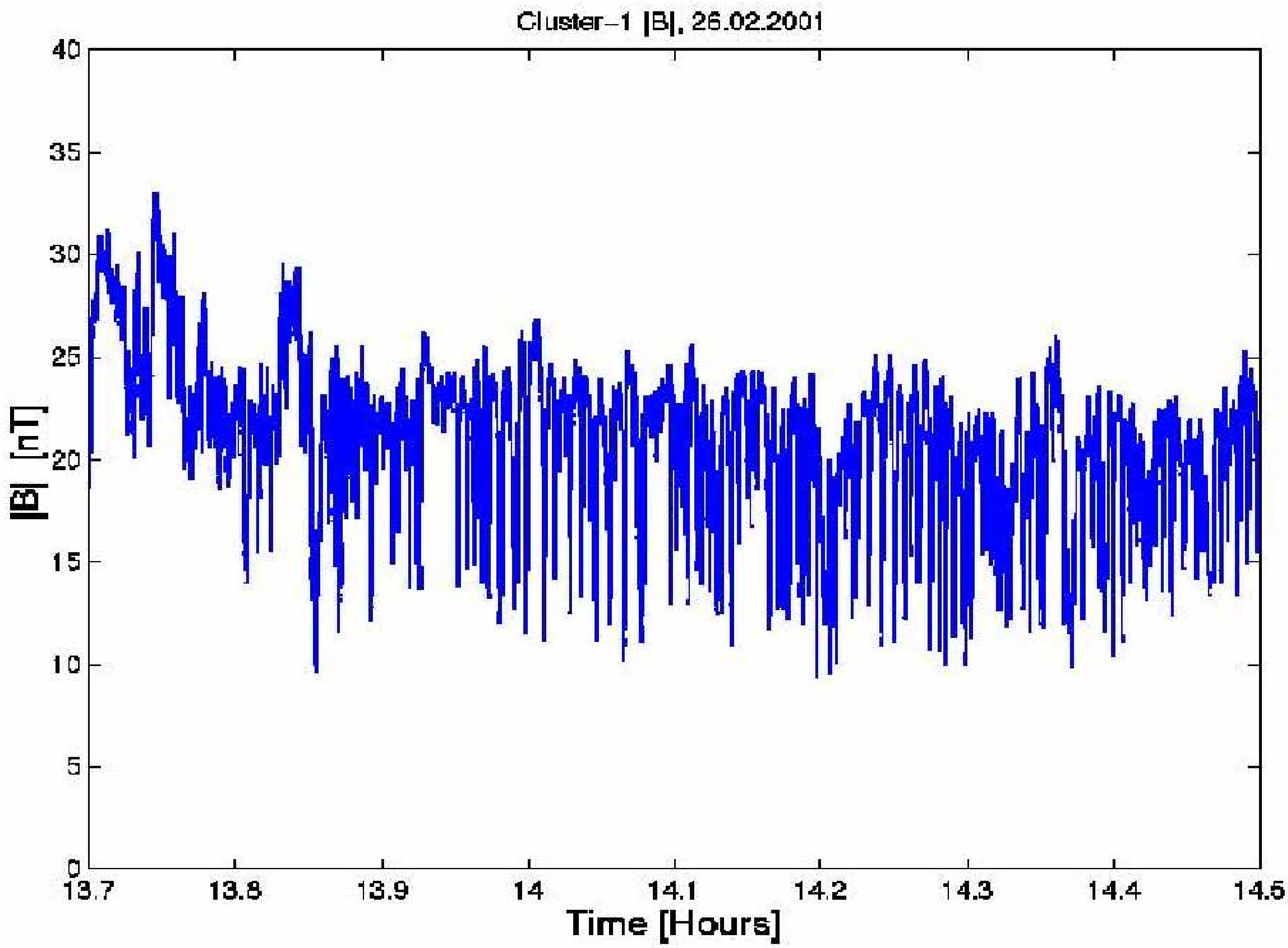
Introduction

- The mirror instability was found by Vedenov and Sagdeev with the help of kinetic description (1957 but published in 1958) using the expansion $\omega/\omega_{ci} \ll 1$. The growth rate:

$$\gamma = |k_z| v_{T_\parallel i} \frac{2\beta_\parallel}{\sqrt{\pi}\beta_\perp} \left[\frac{\beta_\perp}{\beta_\parallel} - 1 - \frac{1}{\beta_\perp} - \frac{k_z^2}{k_\perp^2} \left(1 + \frac{\beta_\perp - \beta_\parallel}{2} \right) \frac{1}{\beta_\perp} \right].$$

where $\beta_\parallel = 8\pi p_\parallel / B^2$ and $\beta_\perp = 8\pi p_\perp / B^2$, ion distribution function $f(v_\parallel, v_\perp)$ is assumed bi-Maxwellian and electrons cold.

Introduction



Introduction

According to measurements in the Earth magnetosheath (Lucek et.al., (2001)) holes have the form elongated in the mean magnetic field direction with maxima of density and pressure. A typical depth of such magnetic holes is about 20% from the mean magnetic field value, sometimes the depth can achieve 50 %. The characteristic width of such structures is about 2-4 ion Larmor radii with aspect ratio about 7-10. These structures are often associated with development of the mirror instability.

Introduction

- The applicability condition of MI $\gamma/k_z \ll v_{T_\parallel i}$ means that

$$\varepsilon = \frac{2\beta_\perp}{2+\beta_\perp-\beta_\parallel}\left(\frac{\beta_\perp}{\beta_\parallel}-1-\frac{1}{\beta_\perp}\right) \ll 1.$$

Main equations for the mirror modes in the drift approximation

The drift kinetic equation for ions:

$$\frac{\partial f}{\partial t} + v_{\|}\mathbf{b}\cdot\nabla f - \mu\mathbf{b}\cdot\nabla B\frac{\partial f}{\partial v_{\|}} = 0.$$

where $\mu = v_\perp^2/2B$ is the adiabatic invariant, $\mathbf{b} = \mathbf{B}/B$ (electric drift and parallel electric field are not essential).

Both pressures $p_{\|}$ and $p_\perp$ are given by the integrals:

$$p_{\|} = mB\int v_{\|}^2 f d\mu dv_{\|} d\varphi \equiv m\int v_{\|}^2 f d^3v,$$

$$p_\perp = mB^2\int \mu f d\mu dv_{\|} d\varphi \equiv \frac{1}{2}m\int v_\perp^2 f d^3v.$$

Main equations for the mirror modes in the drift approximation

The equation for the balance of forces (transverse components):

$$0 = \Pi \left\{ -\nabla \left(p_\perp + \frac{B^2}{8\pi} \right) + \left[1 + \frac{4\pi}{B^2} \left(p_\perp - p_\parallel \right) \right] \frac{\mathbf{B} \cdot \nabla \mathbf{B}}{4\pi} \right\}$$

where the first term in the r.h.s. describes action of magnetic and perpendicular pressures, the second term is responsible for magnetic lines elasticity, $\Pi_{ik} = \delta_{ik} - b_i b_k$. Here the inertia term is omitted. It is small in both linear and nonlinear regimes.

Two Maxwell equations:

$$\nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}.$$

Behavior of the growth rate near threshold in the bi-Maxwellian case

Near threshold the instability is saturated due to the FLR effect :

$$\gamma = |k_z| \frac{v_{T_\parallel} \beta_\parallel}{\sqrt{\pi} \beta_\perp} \left[\frac{\beta_\perp}{\beta_\parallel} - 1 - \frac{1}{\beta_\perp} - \frac{k_z^2}{k_\perp^2 \beta_\perp} \left(1 + \frac{\beta_\perp - \beta_\parallel}{2} \right) - \frac{3}{4\beta_\perp} k_\perp^2 \rho_i^2 \right]$$

In this case $k_\perp \propto \sqrt{\varepsilon}$, $k_z / \, k_\perp \propto \sqrt{\varepsilon}$ and $\gamma \sim \varepsilon^2$.

Quadratic nonlinearities

Assuming

$$\frac{\widetilde{B}_z}{B_0} = \varepsilon \frac{2\beta_\perp}{1+\beta_\perp}\left(1 + \frac{\beta_\perp - \beta_\|}{2}\right) U,$$

the multi-scale expansion yields

$$\frac{\partial U}{\partial T} = \widehat{K}_z \left[\left(1 - \Delta_\perp^{-1}\frac{\partial^2}{\partial Z^2} + \Delta_\perp\right) U - 3U^2\right],$$

where Here $\widehat{K}_z = -\widehat{H}\frac{\partial}{\partial Z}$ and $\widehat{H} f(Z) = \frac{1}{\pi} VP \int_{-\infty}^{\infty} \frac{f(Z')}{Z'-Z} dZ'$ is the Hilbert transform.

The 3D model

The obtained equation possesses one remarkable property:

$$\frac{\partial U}{\partial T} = -\widehat{K}_z \frac{\delta F}{\delta U}$$

where

$$F = \int \left[-\frac{1}{2} U^2 + \frac{1}{2} U \Delta_\perp^{-1} \frac{\partial^2}{\partial Z^2} U + \frac{1}{2} \left(\nabla_\perp U \right)^2 + U^3 \right] d^3 R.$$

F has the meaning of the free energy or the Lyapunov functional.

The 3D model

This quantity can not grow but only decrease in time:

$$\frac{dF}{dT} = \int \frac{\delta F}{\delta U} \frac{\partial U}{\partial T} d^3 R = - \int \frac{\delta F}{\delta U} \widehat{K}_z \frac{\delta F}{\delta U} d^3 R \leq 0.$$

dF/dT vanishes on the stationary localized solutions:

$$\frac{\delta F}{\delta U} \equiv \left(1 - \Delta_\perp^{-1} \frac{\partial^2}{\partial Z^2} + \Delta_\perp \right) U - 3U^2 = 0.$$

Such solutions are possible at $\varepsilon < 0$ but unstable; above threshold, $\varepsilon > 0$, they are absent.

The 3D model

Thus, the derivative dF/dT is strictly negative and respectively F decreases monotonically in time, becoming more negative. For small amplitudes such regime provides by the first term. At the nonlinear regime negativeness of F provides by the last term, i.e., $\int U^3 d^3R$. The latter means that at the nonlinear stage

$$\int U^3 d^3R < 0$$

that corresponds to the formation of magnetic holes.

This process has blow-up behavior. It is possible to show that that $F|_{t=0} < 0$ is the sufficient condition for collapse.

Quasi-linear effects

- The growth rate with account of FLR effect for arbitrary distribution function

$$\gamma_k = \sqrt{\frac{2}{\pi}}|k_\parallel|\tilde{v}\Big(\Gamma - \frac{3}{2}\tilde{r}^2 k_\perp^2 - \frac{k_\parallel^2}{k_\perp^2}\chi\Big),$$

where $\Gamma = \beta_\Gamma - \beta_\perp - 1$,

$$\beta_\perp = \frac{mn}{p_B}\int \frac{v_\perp^2}{2} f d^3v, \quad \text{and} \quad \beta_\Gamma = -\frac{mn}{p_B}\int \frac{v_\perp^4}{4}\frac{\partial f}{\partial v_\parallel^2} d^3v.$$

Quasi-linear effects

Here $p_B = B_0^2/8\pi$ is the magnetic pressure, Ω the ion gyrofrequency,

$$\chi = 1 + \frac{1}{2}(\beta_\perp - \beta_\|) \ \text{ with } \ \beta_\| = \frac{mn}{p_B}\int v_\|^2 f d^3v,$$

$$\tilde{v}^{-1} = -\sqrt{2\pi}\frac{mn}{p_B}\int \frac{v_\perp^4}{4}\delta(v_\|)\frac{\partial f}{\partial v_\|^2} d^3v$$

$$\tilde{r}^2 = -\frac{mn}{24 p_B}\frac{1}{\Omega^2}\int \left(v_\perp^6 \frac{\partial f}{\partial v_\|^2} + 3 v_\perp^4 f \right) d^3v.$$

Quasi-linear effects

The asymptotic model for arbitrary distribution function reads

$$\partial_t b = \sqrt{\frac{2}{\pi}} \tilde{v} \left(-\mathcal{H}\partial_z\right) \left(\Gamma b + \frac{3}{2}\tilde{r}^2 \Delta_\perp b - \chi \frac{\partial_z^2}{\Delta_\perp} b - \Lambda b^2\right),$$

where $b = \delta B_z(r,t)/B_0$,

$$\Lambda = \beta_\Lambda - 2\beta_\Gamma + \frac{\beta_\perp}{2} + \frac{1}{2} \ \text{with} \ \beta_\Lambda = \frac{mn}{p_B} \int \frac{v_\perp^6}{8} \frac{\partial^2 f}{\partial (v_\parallel^2)^2} d^3 v,$$

Quasilinear equation

Near the instability threshold quasi-linear diffusion for ions mainly takes place along magnetic field (Shapiro & Shevchenko, 1964):

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial v_\parallel} D_{\parallel\parallel} \frac{\partial f}{\partial v_\parallel}$$

with

$$D_{\parallel\parallel} = v_\perp^4 \sum_k \frac{|b_k|^2}{4} \frac{\gamma_k k_\parallel^2}{k_\parallel^2 v_\parallel^2 + \gamma_k^2}.$$

Quasi-linear (2+1) diffusion

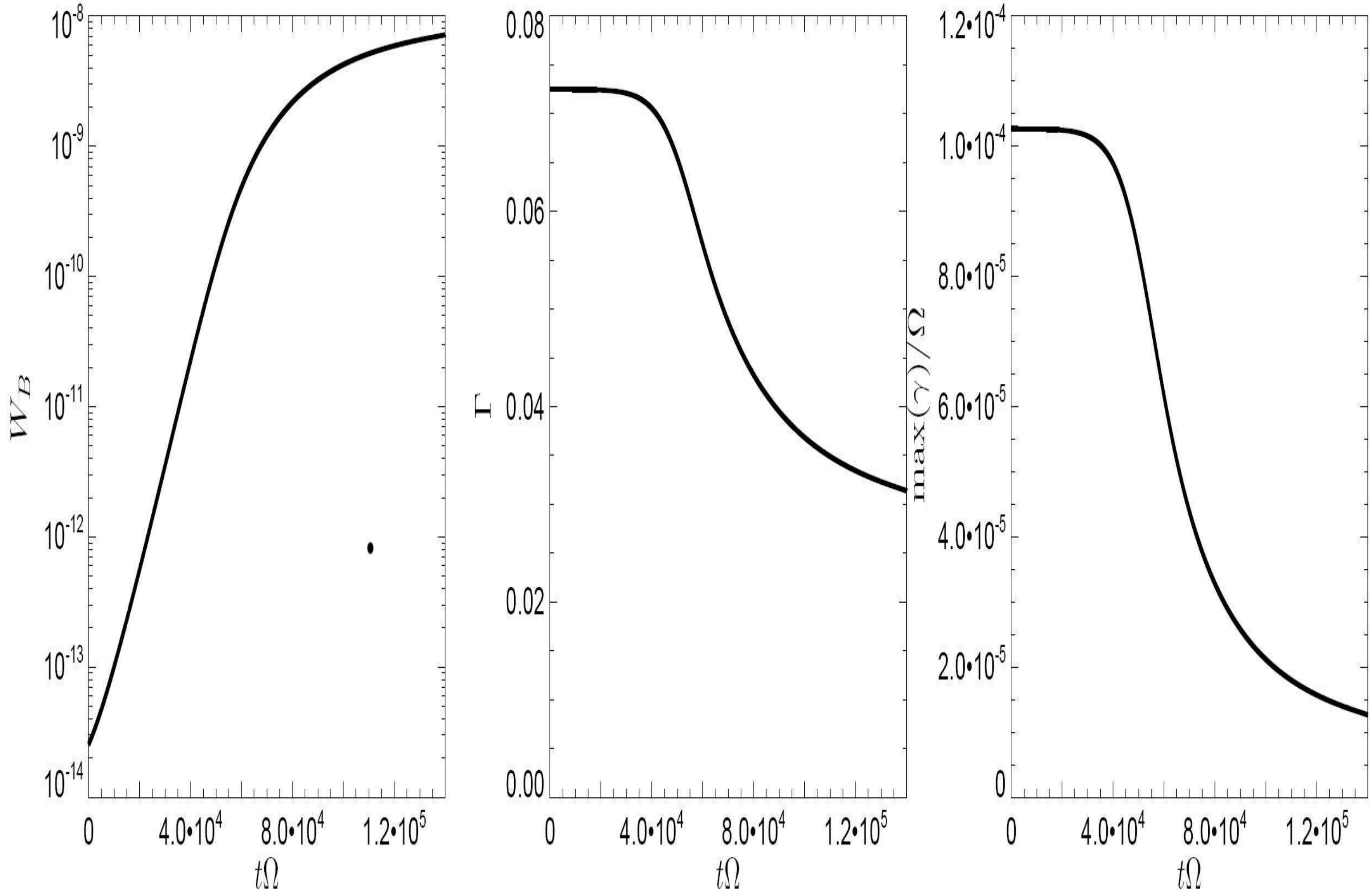


Figure 1. Results of the QL simulation: (top) fluctuating magnetic energy $W_B = \sum_k |b_k|^2$, (middle) distance from threshold Γ, (bottom) maximum growth rate $\max(\gamma)$ as functions of time.

Quasi-linear (2+1) diffusion

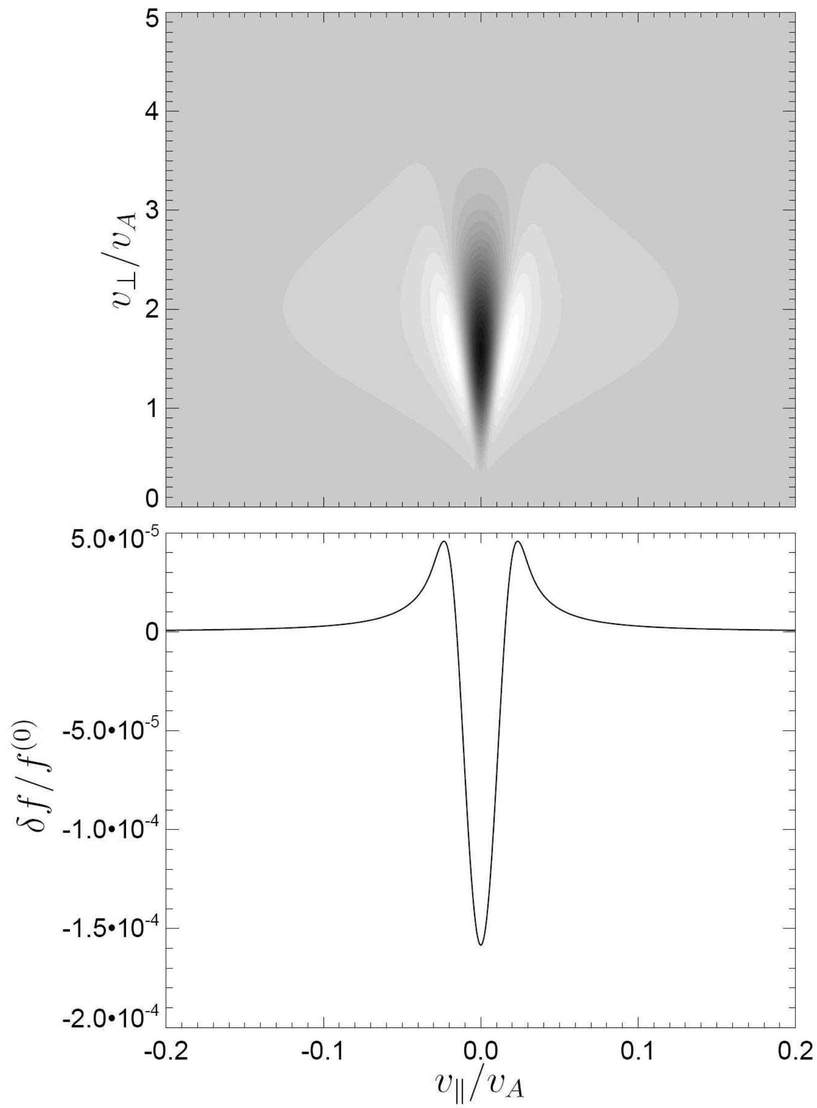


Figure 2. Results of the QL simulation at $t = 1.4 \cdot 10^5/\Omega$: (top) Gray scale plot of $v_\perp \delta f$ as a function of $v_\parallel$ and $v_\perp$. Black corresponds to negative values and white to positive ones; (bottom) Profile of $\delta f/f^{(0)}$ as a function of $v_\parallel$ at $v_\perp = 2v_A$.

Particle-in-cell and 'Vlasov' (2+1) simulations

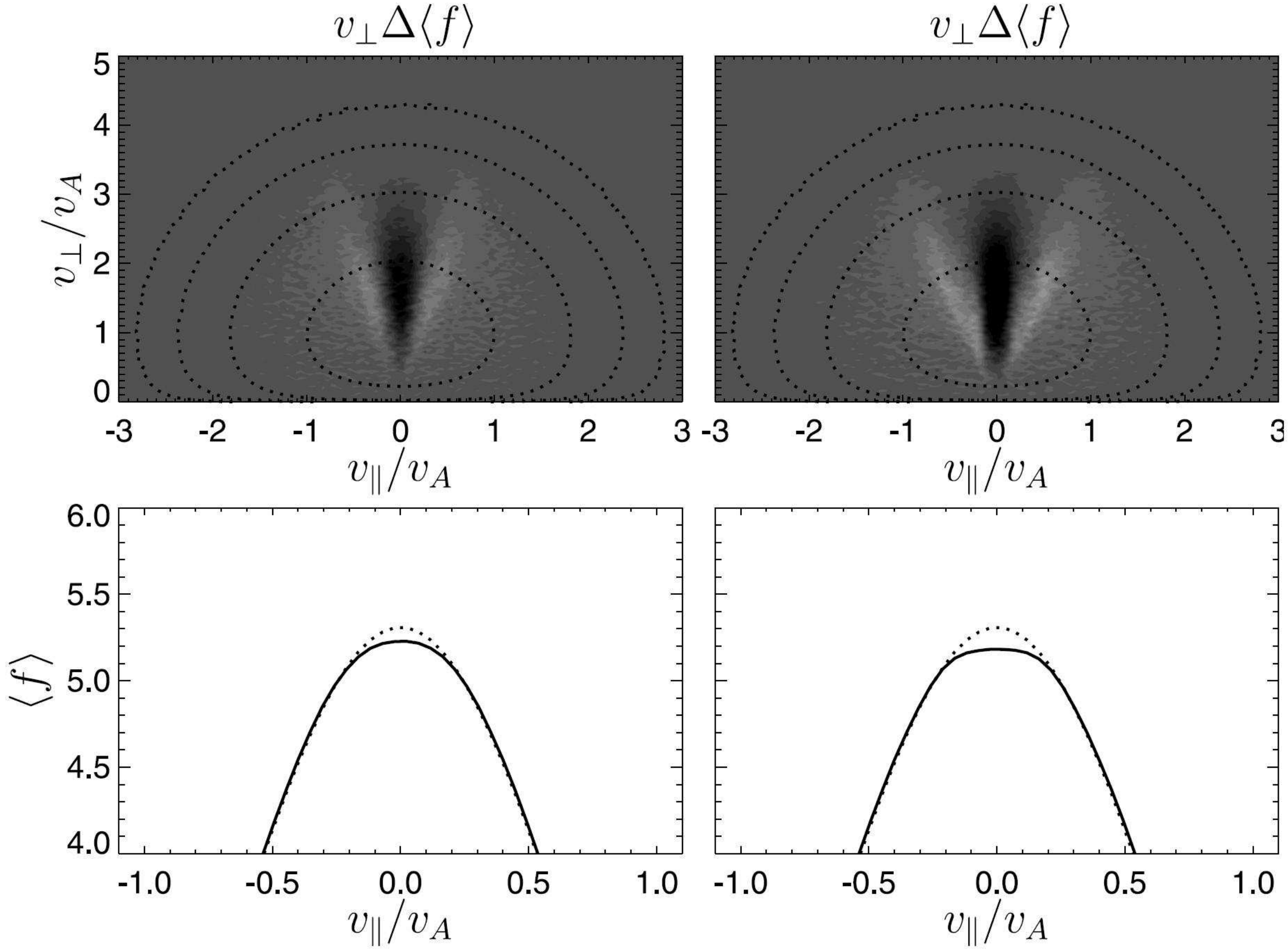


Figure 4. Same conditions as Figure 1. Simulation results at times (left) $t = 2000/\Omega_p$ and (right) $t = 10{,}000/\Omega_p$: (top) Gray scale plots of the proton distribution variation $v_\perp \Delta\langle f\rangle$ (black corresponds to negative values and white to positive ones). Dotted lines correspond to the contours of the initial condition $v_\perp f^{(0)}$. (bottom) Profiles (solid line) of the proton distribution function $\langle f\rangle$ integrated over $v_\perp$, together with the initial profile (dotted line).

Blowing-up formation of 1D humps

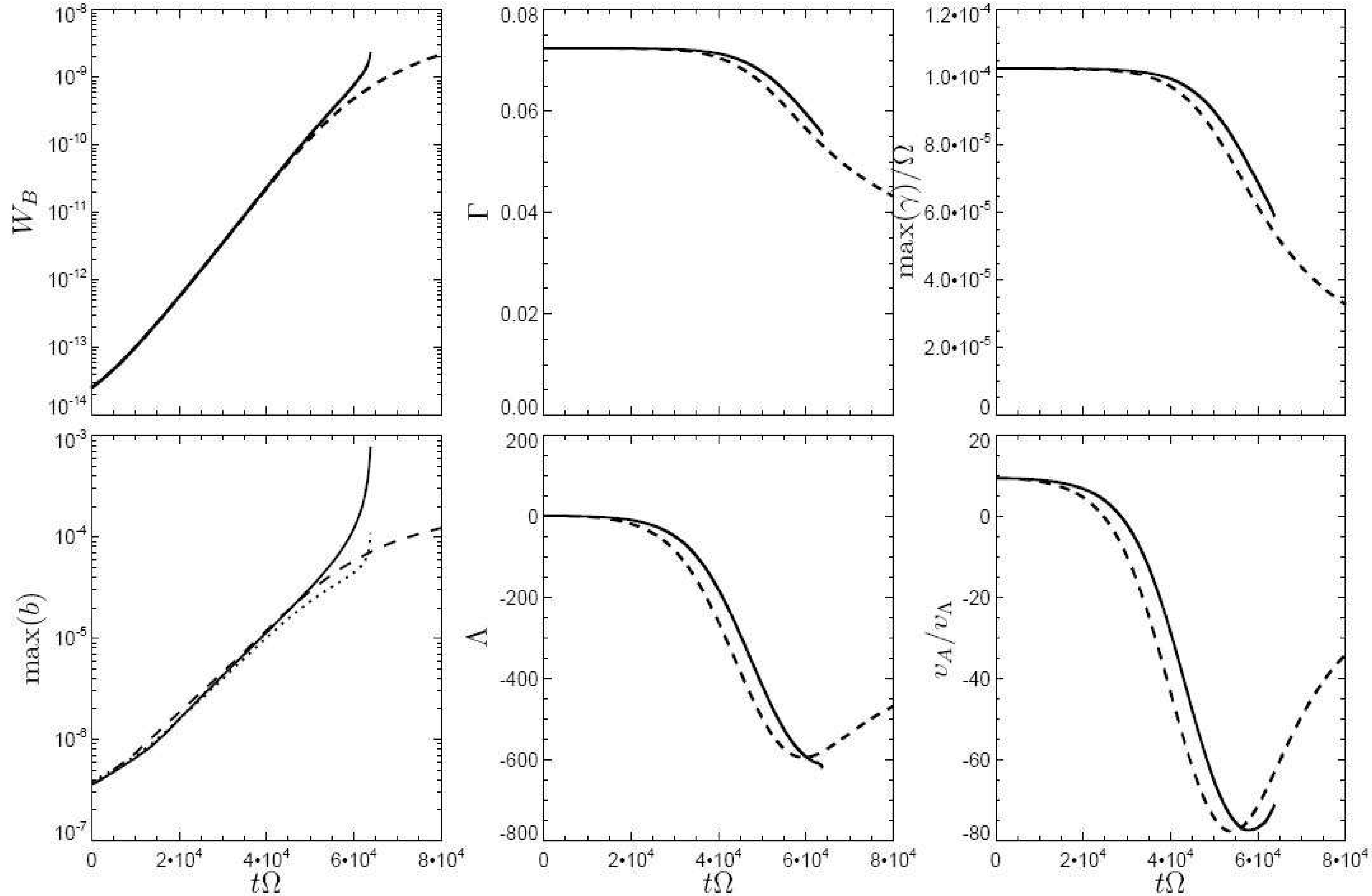


Figure 3. Results of the simulation of equation (16): solid lines show the time evolution of(from left to right, from top to bottom) $W_B = \sum_k |b_k|^2$, Γ, maximum of γ_k, maximum of the magnetic fluctuations $b(x)$, Λ and v_Λ^{-1}. For comparison, dashed lines show the evolution in the QL model. The dotted line in the left-bottom panel refers to the evolution of the maximum of $-b(x)$, as predicted by equation (16).

Conclusion

- Quasi-linear evolution of the mirror instability was investigated by direct integration of the corresponding diffusion equation. The resulting flattening of the distribution function is in good agreement with the early time results of Vlasov-Maxwell simulations.
- A dynamical model was presented that reproduces the formation of mirror structures observed at later times. It provides a possible mechanism for the formation of magnetic humps in a mirror unstable plasma, as revealed by satellite measurements.

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